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ABSTRACT

Reported is research conducted as a part of the Project on Analysis of Mathematics Instruction. The study had two main purposes: to test the feasibility of teaching topics in probability and statistics to a class of sixth grade students; and to construct a set of instructional materials and procedures in probability and statistics for sixth graders. A unit of instruction was prepared and the order in which behavioral objectives were to be taught was determined from a content outline and a task analysis. The results of the study support the feasibility of teaching most of the topics covered in the unit to average and above average sixth graders. The study also lends support to the use of the systems developmental model employed for developing curriculum materials. Part II includes Appendix A which contains lesson plans, exercises, and quizzes used in the study. (FL)

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No. 105 (Part II) and Appendix A (printed in two part

A STUDY OF PARTS OF THE DEVELOPMENT OF A UNIT IN PROBABILITY AND STATISTICS FOR THE ELEMENTARY SCHOOL

Report from the Project on
Analysis of Mathematics Instruction

U.S. Office of Education Center No. C-03 Contract OE 5-10-154

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A STUDY OF PARTS OF THE DEVELOPMENT OF A UNIT IN PROBABILITY AND STATISTICS FOR THE ELEMENTARY SCHOOL

Report from the Project on
Analysis of Mathematics Instruction

By Jack L. Shepler

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Wisconsin Research and Development
Center for Cognitive Learning
The University of Wisconsin
Madison, Wisconsin
November 1969

This Technical Report is a doctoral dissertation reporting research supported by the Wisconsin Research and Development Center for Cognitive Learning. Since it has been approved by a University Examining Committee, it has not been reviewed by the Center. It is published by the Center as a record of some of the Center's activities and as a service to the student. The bound original is in The University of Wisconsin Memorial Library.

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STATEMENT OF FOCUS

The Wisconsin Research and Development Center for Cognitive Learning focuses on contributing to a better understanding of cognitive learning by children and youth and to the improvement of related educational practices. The strategy for research and development is comprehensive. It includes basic research to learning and about the processes of instruction, and the subsequent development of research-based instructional materials, many of which are designed for use by teachers and others for use by students. These materials are tested and refined in school settings. Throughout these operations behavioral scientists, curriculum experts, academic scholars, and school people interact, insuring that the results of Center activities are based soundly on knowledge of subject matter and cognitive learning and that they are applied to the improvement of educational practice.

This Technical Report is from Phase 2 of the Project on Prototypic Instructional Systems in Elementary Mathmatics in Program 2. General objectives of the Program are to establish rationale and strategy for developing instructional systems, to identify sequences of concepts and cognitive skills, to develop assessment procedures for those concepts and skills, to identify or develop instructional materials associated with the concepts and cognitive skills, and to generate new knowledge about instructional procedures. Contributing to the Program objectives, the Mathematics Project, Phase 1, is developing and testing a televised course in arithmetic for Grades 1-6 which provides not only a complete program of instruction for the pupils but also inservice training for teachers. Phase 2 has a long-term goal of providing an individually guided instructional program in elementary mathematics. Preliminary activities include identifying instructional objectives, student activities, teacher activities materials, and assessment procedures for integration into a total mathematics curriculum. The third phase focuses on the development of a computer system for managing individually guided instruction in mathematics and on a later extension of the system's applicability.

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ABSTRACT

From a content outline and a task analysis the behavioral objectives for a unit of instruction in probability and statistics for sixth-grade students and the order in which objectives would be taught were determined. An instructional analysis of the unit was undertaken to select or develop materials and procedures for teaching the unit.

Data from a pilot study conducted in the fall of 1969 were used to identify a set of nine lessons that could be formatively evaluated to test the feasibility of the instructional analysis. The lessons were used to teach a class of 25 sixth-grade students of average to above average ability. The topics developed through experiments, games and exercises were subjective probability notions, empirical probability, counting techniques, a priori probability including simple and compound events, and comparison of two events using probability.

On the basis of the overall pretest and posttest the instructional treatment was generally successful. The pretest percentage was 37.9% and the posttest percentage was 92.8% with all 72 items successful for 11 of the 14 measured objectives. Instruction was unsuccessful in getting students to specify the estimated probability; number the outcomes of an event; and estimate the probability successful for these three objectives because of a lack of stress and practice. Two learning hierarchies were also tested. One hierarchy was validated and the other was not. The results of the study support the feasibility of teaching most of the included topics in probability and statistics to average and above average sixth-grade students given high quality of teaching. The study lends support to the use of the systems developmental model employed in this study for developing curriculum materials for the schools, especially when used in conjuction with Bloom's "Mastery Learning" techniques.

APPENDIX A

JOURNAL OF STUDY
(INCLUDING LESSON PLANS, EXERCISES, QUIZZES, AND COMMENTS)

DISCUSSION OF PRETESTING AND PREINSTRUCTION

Monday (3/3), Tuesday (3/4), Wednesday (3/5)

The formative experimental unit at Waunakee began with two days of pretesting. The testing was conducted in the classroom that was to be used for instruction for the unit.

On the first day (Monday, March 3) part A of the pretest was administered. The directions for the test were read to the class. Models of spinners, marbles and coins were shown to the class. The children were given unlimited time to do the test. Most of the class finished the test in 20 to 25 minutes. One boy took 35 minutes.

On March 4 (Tuesday) the second half of the pretest was administered. The directions were read again. This testing session lasted 25 minutes. The remaining twenty-five minutes were spent on reading a bar graph. Although the students had not made bar graphs, they were somewhat familiar with them. The lesson was conducted as planned with no problems. The first 30 minutes of Wednesday's session was spent in collecting data and having the children construct graphs of the data.

Lesson A

INTERPRETING A BAR GRAPH (A PREREQUISITE BEHAVIOR)

Objective:

1. Interpret a bar graph.

Materials to be used:

1. Transparency #1.
2. Handout sheet.
3. Overhead projector.

New Vocabulary:

bar graph, scales, vertical scale, horizontal scale.

Method of Presentation:

Using an overhead projector, show the children Transparency #1. Cover the bottom part with a sheet of paper so they are able to see the data table only. Explain that the table at the top of the page tells how many TV sets there are per 1,000 people in six different countries. For example, in France there are 110 TV sets for every 1,000 people. These questions may be asked in order to familiarize the children with the table:

1. Which country has the least number of TV sets per 1,000 people?
2. Which country has the most?

Show the children the bar graph by removing the paper. Make sure the table is still in view. Explain that this is a bar graph and simply tells you the same thing which the above table told you. These questions may be asked:

1. Where does the bar graph show us the number of TV sets per 1,000 people?
2. Is it clearly labeled?
3. What else does the bar graph need to tell us that the table told us?
4. Does it clearly show the countries?
5. Can you tell by looking at the graph which country has the most TV sets per 1,000 people?
6. Is that what we decided by just looking at the data at the top?
7. Which country has the least?
8. By looking at the bar graph (cover the table) can you tell me which countries had fewer than 150 TV sets per 1,000 people? (uncover the table.)
9. Now look at the table and see if you were correct.
10. When you were trying to compare the data, was it easier to use the table or the bar graph?

"Now I would like you to look at some data I gathered about your town and put on a bar graph." Hand out copies of the bar graph on families in the Waunakee Telephone Directory. Discuss these questions before children are asked to answer the questions at the bottom of the bar graph.

1. What kind of information is given along the horizontal scale?
2. Along the vertical scale?
3. Are both scales clearly marked?
4. Exactly how many people are listed under the letters A B?
5. Do you suppose that is an exact figure?
6. If you went to the telephone directory and counted the number of families listed under the letters A and B, do you think it would come to exactly 150 families?
7. What would you have to do in order to show on a bar graph the exact number of families listed in the Waunakee Telephone Directory?

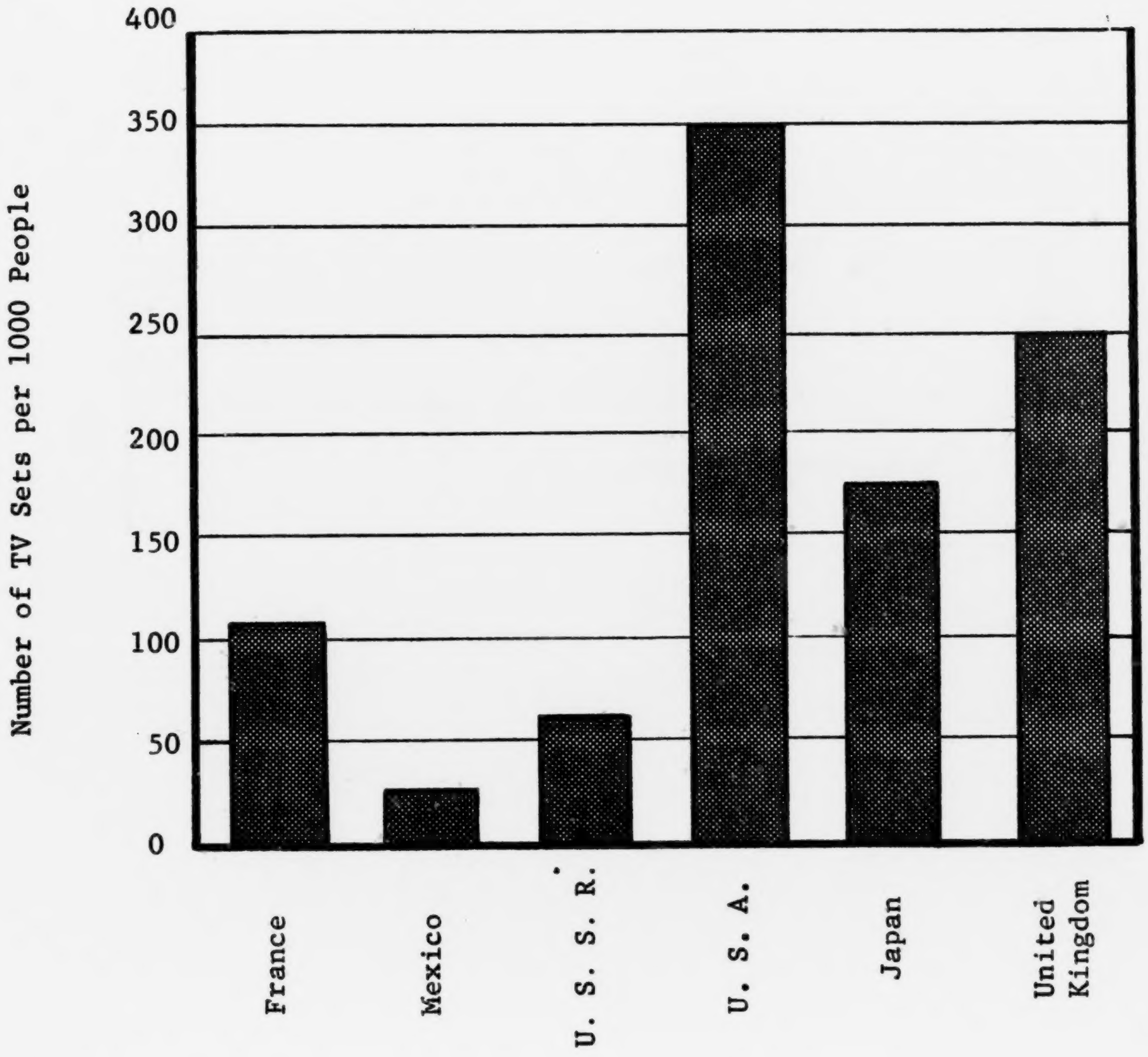
Ask if there are any questions about the bar graph. Ask children to answer the questions at the bottom.

[Comment:

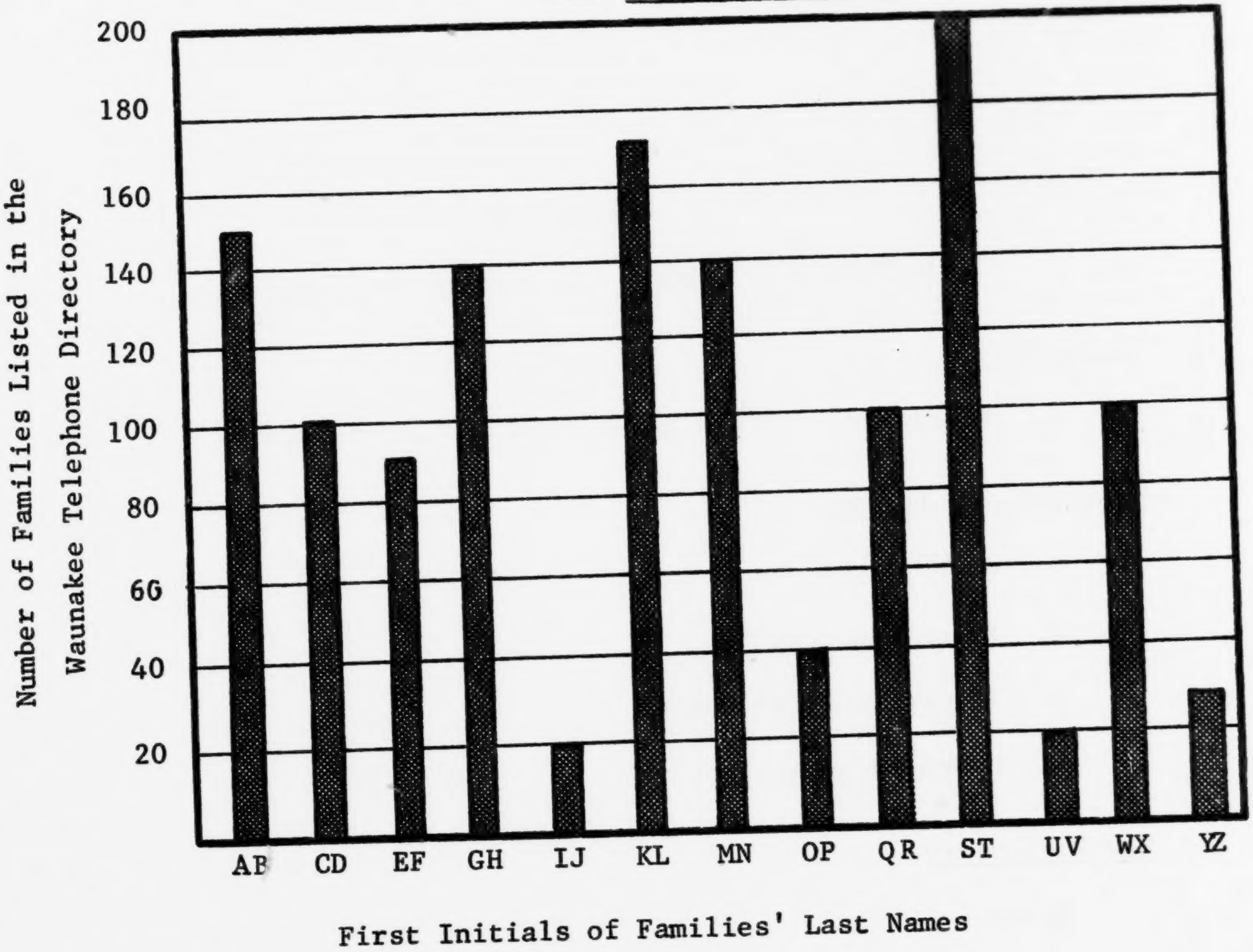
The exercise was collected at the end of the period and graded. (24s/25s were graded as Masters.) The exercise was handed back and discussed on Thursday.]

TRANSPARENCY #1

Country	Number of TV Sets per 1000 People
France	110
Mexico	30
U. S. S. R.	60
U. S. A.	350
Japan	175
United Kingdom	250



Name ________________ Date __________



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1. *Which letters have less than 80 families listed?* 1. ____________________

2. *Which letters have more than 160 families listed?* 2. ____________________

3. *Which letters have the most families listed?* 3. ____________________

4. *Which letters have about 100 families listed?* 4. ____________________

5. *Do the letters "MN" have more than or less than 150 families listed?* 5. ____________________

6. *List the letters in the order of the number of families listed. Begin with the least number.* 6. ____________________

____

Lesson B

PLOTTING A BAR GRAPH (A PREREQUISITE BEHAVIOR)

Objective:

1. To construct a bar graph from the given data.

Materials to be used:

1. Graph paper.
2. Ruler and pencil.
3. Transparency #2 and #3.
4. Overhead projector.

New Vocabulary:

None.

Method of Presentation:

"Yesterday I showed you two bar graphs I had made. Today we're going to collect some data in class and you can make a bar graph. We'll make a bar graph showing how many of you were born in each month of the year. I'll record the data up here on the overhead projector." Ask each child during which month he was born and record it by putting a tally mark for each one. Explain that this is gathering and recording data. Total the number of children born in each month and clearly show on the overhead as this is the information they need to graph. Ask the children if they think the data is accurate and whether it was gathered and recorded correctly. Show the children a copy of a piece of graph paper.

Put Transparency #3 (a grid) on the overhead. While the graph paper is being distributed, transfer the totals to the chalk board, large enough so all the children can clearly see the data.

Draw in the scale lines on the overhead and explain to the children that this has already been done on their paper. Talk about what information you want to show on the graph and where it should go. "Which information should we put on the horizontal scale?" You may need to show previous two graphs as examples. (In order to have the bars vertical, the names of the months need to be on the horizontal scale.) Children should label the horizontal scale Months of the Year. "Next we must decide how much space each month needs on the horizontal scale. How many months are there? How much space do we have, or how many divisions do we have along the bottom? How many spaces can we use for each month? Now you can draw a short line every three spaces to show where each month will go." Show on the overhead first. "Finally you can write the name of each month in the correct space." Show again on the overhead. "Now we can label the vertical scale. Are there any suggestions?" The label should be Number of Children. "From the data, what is the maximum number we're going to need on this scale? About how many divisions do we have? How many divisions can we allow for each child? Once we decide that we can draw small lines again to show each division, then these can again be labeled. What number do we start with?" 1) If the response is zero, ask "Where do we put the zero?" If they don't know ask them to recall the two previous graphs. If this still doesn't help show them the graphs. 2) If the response is one, point out a month in which no children were born. Ask "How are you going to show that?" You may need to show them the two previous graphs. Once the divisions have been decided upon the numerals can be put on the vertical scale.

"Now all we need to do is transfer our data from the table on the board to our graphs. Starting with Jan., how many children were born during this month? Now find and put a small mark above Jan. at that number. You can either fill the bar in now or wait until you have completely finished and then fill all the bars in at once. Remember to double check your bar graph, so you don't record Feb. results over Mar. Are there any questions?" Children can proceed on their own, with teacher helping those who need personal direction.

Comment:

After this activity was completed, the data for "the number of children in your family" was collected by the teacher and placed on the board. The students were given a bar graph with the vertical and horizontal axes already drawn (to save time). However, the scales had not been marked. The students were instructed to draw a bar graph of the data.

TRANSPARENCY #2

Jan.

= 0

Feb.

1111 = 4

Mar.

1111 = 4

April

11 = 2

May

11 = 2

June

111 = 3

July

1 = 1

Aug.

11 = 2

Sept.

= 0

Oct.

1 = 1

Nov.

1 = 1

Dec.

1111 = 4

#3

TRANSPARANCY #3

Lesson B

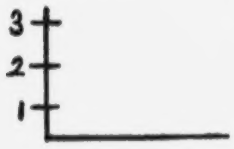
COMMENTS ON CLASS EXERCISE ON BAR GRAPHING

The graphs that the children made of the number of children having birthdays on the same month were done together. (Data recorded on Transparency #2.) The children were then asked how many children there were in their family. The data was collected and recorded on the board. The children were asked to draw a bar graph of the data on the graph paper provided. The graphs were collected and graded. Only 6s/25s had perfect graphs.

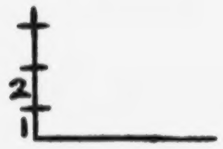
The common mistakes were in not doing the following:

1. Labeling of numbers on the vertical axis. Many did not place the number opposite the mark on the vertical scale

[e.g.



For example, they would label it:

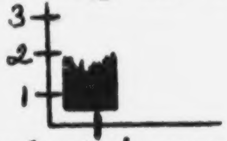


This then caused problems as to where the bar should terminate.)

2. Naming the vertical scale. (Some forgot to do this.)

3. Starting the bars at the zero position.

[e.g.



]

The students' papers were returned on Friday (3/7), and the common mistakes were discussed. The students were asked to correct their mistakes. All were able to do this. One student (subject 2) had particular problems and was confused by the vertical and horizontal scales and how to plot values. The author worked with her for ten minutes while the other students worked on graphing the experiments of Lesson 3.

Lesson 1

INTRODUCTION TO PROBABILITY

Objectives:

The child should be able to:

1. Identify an experiment (1-D).
2. Identify an outcome of an experiment (1-D).
3. Identify the set of possible outcomes of an experiment (1-D).
4. Identify an instance of certainty.
5. Identify an instance of uncertainty.
6. Identify an instance of impossibility.
7. Distinguish between certain, uncertain (possible) and impossible events.
8. Identify equally likely outcomes of an experiment.
9. Identify unequally likely outcomes of an experiment.
10. Identify experiments which are equivalent.
11. Identify which of two or more events is more likely or less likely.

Prerequisite Behaviors:

None.

Materials to be used:

(Code: R-Red, B-Blue, Y-Yellow)

1. Spinner 1/2R, 1/2B.
2. Spinner 1/3R, 1/3B, 1/3Y.

3. Spinner 3/4B, 1/4R.
4. Marbles 1 R, 1 W; 1 R, 1 B, 1 Y; 3 W, 1 R.
5. Coin.
6. Dice.
7. Workbook materials from SMSG.

Vocabulary:

chance, certain, uncertain, more likely, just as likely (equally likely), outcome, possible, probable, probably, impossible, probability, experiment.

Introduction to Unit:

You probably have heard statements such as:

"The chances are good that we will win the basketball game tonight."

"It probably will rain today."

Chance, probably, probability, etc. are some of the words which are very important in what we are going to study for the next few weeks. The name of the subject we will be studying is probability--the mathematics of the uncertain.

We will be using spinners, dice, and games to try to understand how to use probability and thus find out what probability means.

Everyone will have the opportunity to make an A for this unit. In fact, we hope everyone does. We will tell you exactly what you've got to do to earn an A. I believe everyone in this class is capable of earning such a grade and besides, we think you can have fun doing it. We hope, in other words, that everyone will master completely the topics in the lessons.

The test you took Monday and Tuesday involved many of the concepts I want you to learn. They measured what you knew about the following principles:

(To be shown on an overhead or a poster--not to be read.)

Comment:

The goals were written on large poster paper.

Lesson 1 1. Tell when something is certain, uncertain, or impossible.

Lesson 2 2. Count outcomes of an experiment.

Lessons 2 & 6 3. Compute the probability of an event.

Lessons 4, 5 & 6 4. Compute the probability of the certain event and of the impossible event.

Lesson 7 5. Tell when two fractions are equal. If they are not equal then tell which is the bigger fraction of the two.

Lessons 7 & 8 6. Tell if two games make you just as likely to win. If they don't, tell which game is most likely to make you a winner.

Lessons 4 & 9 7. Understand what is meant by the probability of an event.

Lessons 2 & 8 8. Carry out an experiment to estimate the probability of an event.

Lesson 9 9. Compute the estimated probability of an event.

Lesson 2 10. Record and organize data.

Lesson 2 11. Construct a bar graph and interpret the results.

Beside each of the above principles are the lessons which will teach these. We will tell you each day which lesson we are on. Some lessons will take more than one day to complete.

Here is the plan we are going to use to get you to be master learners of these skills.

Each lesson has a set of problems which we would like you to do to the best of your ability. After they are completed we will normally grade you as to whether you are a master or a nonmaster learner depending on how well you do. If you are a master learner, you go on. If you are a nonmaster learner, you will be given a set of instructions to do some further work to enable you to become a master learner. If you do this successfully, you will be a master learner for that topic. We hope that everyone will become a master learner for every topic. If you are classified as a master learner 90 per cent of the time and do well on the final test, you will receive a diploma which states that John Smith was a master learner of probability. You will also receive an A for the unit.

Comment:

Children at this point asked if the grades would go on their report card. The teacher said she didn't know what the classroom teacher wanted to do, but that she would check. (The grades did not go on their report card.)

Method of Presentation:

one red marble
and one yellow
marble

"Now let's get back to talking about chance or probability." Ask a student to come to the front of the room. As you put a red marble and a yellow marble in the box, ask the class to guess which marble the student will pick from the box.

Discuss their expectations. They'll probably make statements such as:

> "It's fifty-fifty." "Chances are even." "It's equal either way." "It's one out of two." "Who can tell?" "You can't be sure." "He's as likely to get red as yellow."

Comment:

Children said, "They're equal," and "Red is as likely as yellow."

Comment that these statements are concerned with chance; that there are many events in life which are uncertain and this is one of them. Discuss what they might expect if each of them came up to draw a marble or if John chose several times, returning the marble drawn to the sack each time.

You might ask them to suggest other things in life which are uncertain; for example:

Who will win the World Series?

Will I have ice cream for lunch some time this week?

Will all the pupils in the class be as school next Monday morning?

Ask pupils to suggest things that are certain. For example, day follows night; there are twelve inches in a foot. Then suggest things which are impossible. From their statements, draw out the idea that somethings are more likely to happen than others and that some things are more likely to happen than not. Pupils can respond to such questions as:

Which is more likely, that some pupil will be absent or that the teacher will be absent?

Which is more likely, that you will have cereal for breakfast or that you'll have cereal for lunch?

Which is more likely, that a boy will build a model airplane or that a girl will build a model airplane?

Comment:

The children did not seem to have any problems in recognizing instances of uncertainty, or impossibility. However, specifying instances of certainty was difficult for them. Statements such as "I am going home tonight" and "Mr. Roberts (their regular teacher) is going to come back to this class," were given as instances of certainty. Even after the flaws in these statements were pointed out by members of the class and the teacher, some students still were not willing to accept these flaws as possibilities.

Now show the class that you are putting three red marbles and one yellow one in the box, and ask another pupil to draw out one marble which will be replaced in the box. Pupils can guess again which color they expect to be drawn. Let them discuss reasons for their expectations.

three red marbles
and one yellow
marble

If everyone in the class took turns drawing a marble, what would we expect might be the outcome? Would red be chosen more often than yellow? (We would expect it to be, although it is possible, but not very probable, that red would not be drawn at all.)

Show the spinner with the dial 1/4 red and 3/4 blue. Ask where they have seen spinners before, for what they were used, and why they were used. Bring out that spinners select "by chance" because the person who spins it cannot know in advance where it will stop.

spinner $\frac{1}{4}$ red and $\frac{3}{4}$ blue

If Mary spins this spinner, will the arrow point to red or to blue? Let them discuss their expectations and their reasons; then raise such questions as:

Can you be sure it will point to red? to blue?

Will it point either to red or to blue? (Chances are that it will. However, it could stop on a line.)

Mary spins and the pupils compare their expectations with the outcome. The spinner should be horizontal rather than vertical so that the pointer is not biased. The "fairness" of the spin could be noted, for example:

Would it make a difference if Mary spun the pointer in the other direction? If she pushed the pointer harder? If the pointer were pushed nearer the head? Nearer the tail? (Develop the idea that the spinner is honest.)

Ask the class what they would expect if Mary spun the spinner 100 times. (It would stop on blue more often than on red -- not exactly outcomes of 75 blue and 25 red but something close to this. Or, it will stop on red less often than on blue. It is not expected that pupils will suggest these numbers but they should see that the expectation of blue is greater than of red.) DND*

spinner $\frac{1}{3}$ red $\frac{1}{3}$ blue, and $\frac{1}{3}$ yellow

Show the spinner with the dial $\frac{1}{3}$ red, $\frac{1}{3}$ blue, and $\frac{1}{3}$ yellow. Ask pupils what they think will be the outcome of one spin and other questions similar to those asked before. Do not indicate whether their answers are correct.

How many different possible outcomes are there? (Three, assuming the arrow doesn't stop on a line.) Is there a better chance that the outcome will be blue on one spin with this dial or on one spin with the $\frac{1}{2}$ red and $\frac{1}{2}$ blue dial? (Show both dials.) What would you expect the outcomes to be on 100 spins?

spinner $\frac{1}{2}$ blue and $\frac{1}{2}$ red

Again show the spinner with the dial $\frac{1}{2}$ red and $\frac{1}{2}$ blue. Ask what their expectations would be on one spin; on 100 spins.

Comment:

Children replied: It's uncertain.

---

*Did Not Do

In what way is spinning this spinner similar to the activity of drawing one block from the sack which held one red cube and one yellow cube?

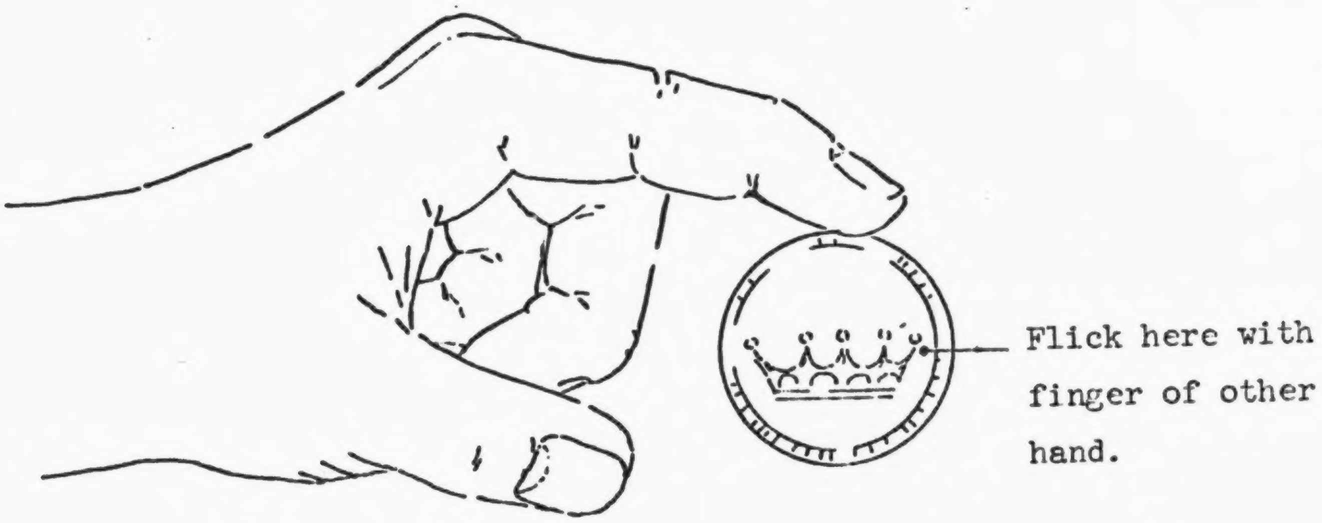
Show the coin, preferably a half-dollar because it is easy for the children to see.

If I toss or spin this coin, will it land with the "head" up?

Discuss which is heads and which is tails; what constitutes a fair toss or spin; that we'll assume the coin doesn't land on its edge so there can only be two possible outcomes--heads or tails; that the coin is honest. In thinking about honest coins, you should be aware that the 1964 penny is NOT honest. If it is spun, on a truly smooth surface, it will land tails up more often than heads up.

Since a penny is lighter than a half-dollar, is it more likely to come up heads than the half-dollar? (In tossing any honest coin, a head is just as likely as a tail.)

DND


Flick here with finger of other hand.

Demonstrate how to toss or spin the coin. Some children find it easier to shake the coin in a paper cup. If a paper is placed on the desk, the noise will be reduced. Let children guess the outcome. Comment that we'll be doing activities using coins.

> Is tossing a coin related to spinning the spinner with the dial $\frac{1}{2}$ blue and $\frac{1}{2}$ red? To drawing one of the two marbles in the box?

Children will probably see the relationship -- "all are equal chances," "one out of two," etc.

DND

die

Show the die, mentioning that it is a cube. Raise questions such as: How many faces does it have? What numbers do the dots represent? If I toss it, is there a fifty-fifty chance that the two dots will show on the top face? What do you think will show when I toss it once?

Toss the die; note that the outcome is the face that is up.

> Do you suppose if I toss the die again, the outcome will be the same?
>
> Would you expect a better chance for a three than a one?
>
> Is a six more likely than a four?

Let pupils discuss their expectations but do not tell them whether or not their ideas are correct. Some pupils may see that each face has an equal chance to be up.

> If a king told you that he'd give you a sack of gold if you could get one of the following outcomes, which would you choose?

1. Blue on a spinner whose dial is 1/2 red and 1/2 blue.
2. a two on one toss of a die.

Let pupils discuss their choices and the reasons for them.

Comment:

The students chose (1) because their chances were better. The teacher felt that the students were convinced that they had made the right decision and she felt no need to discuss it further.

Place on overhead the pictures of the experiments that have been discussed. DND

Ask students if any of these activities are alike. How are they alike? Help students to see that the experiments with 1R and 1Y marbles, the coin, the spinner (1/2R, 1/2B), the die (even, odd), have two equally likely outcomes. Help them to see that there is a similarity between the spinner that is 3/4B, 1/4R, and the experiment with marbles 3R, 1Y. However, for the latter two, the outcomes are not equally likely.

Comment:

When the students were asked, "In what way is spinning this spinner (1/2R, 1/2B) similar to the activity of drawing one marble from the box which holds one red marble and one yellow marble?" they were able to draw the conclusion that both models had two equally likely outcomes. When the students were asked to compare the spinners (1/2R, 1/2B) and (1/4R, 3/4B), they were able to say that the two models had equally likely outcomes while the latter model had outcomes which were not equally likely.

Ask students if they could design an experiment similar to spinning one time the spinner (1/3R, 1/3B, 1/3Y). (If they cannot come up with any ideas--suggest marbles. They should come up with drawing a marble from a box containing three different marbles.) DND

Pass out practice sheet. Collect at the end of the period. If time, play the game with the dice where you take the sums (5,6,7,8,9) and they take (2,3,4,10,11,12). Point out that you have five sums and they have six sums. Have two people recording on the board one point each time you or they win. Go around the room letting each person throw the dice once. Since your probability or winning is twice as great as theirs you should win. Ask students why they think this happened. After a discussion, tell them that the understanding of why this happened is a major goal of future lessons. DND

DISCUSSION OF LESSON 1

Wednesday (3/5)

Approximately 18 minutes were spent on Lesson 1 on Wednesday.

Due to the length of the written lesson, comments concerning the lesson are inserted within the lesson. The parts of the lesson that were not done are marked.

Class discussion centered on demonstration models as directed by the teacher. This was the sole method of instruction used in this lesson. Due to a lack of class time, the exercise accompanying the lesson was assigned as homework.

Since there was not as much time available for this lesson as had been anticipated, at some time in the future lessons having the following objectives need reinforcement:

1. identifying two equivalent models as being equivalent,
2. using the words "outcomes" and "possible outcomes" correctly,
3. listing instances of certainty.

EXERCISE (LESSON 1)*

1. THINKING ABOUT CHANCE

You probably have heard or even made statements such as these:

1. More likely than not we will go to the park on Saturday.
2. Chances are good that we will get to do it.
3. John and Billy have equal chances to win.
4. I am almost certain that I can come to your house after school.

These sentences are alike in one way. They have words and ideas which are used in mathematics. These words and ideas are used in a part of mathematics called probability. In probability, we are interested in things which happen by chance. By using mathematics we can often estimate quite accurately what will probably happen.

We will experiment with such things as coins, spinners, colored blocks, and dice to learn what to expect. Later we will learn what to expect by working with numbers instead of using experiments.

What Do You Know About Chance?

Do you know the answers to these questions?

Who will win the World Series this year?

Will all the members of our class be in school next Monday?

---

*Taken from SMSG's Probability for Intermediate Grades, pp. 1-6.

> How many people in our class will have perfect spelling papers this week?
>
> Will I see a Ford truck on my way home from school this afternoon?

We cannot be certain of the answers to questions such as these because they are chance events. However, there are some things about chance events which we do know.

Some things are more likely to happen than others. For example:

> Which is more likely, that one of the pupils will be absent or that the teacher will be absent?
>
> Which is more likely, that you will have cereal for breakfast or that you will have cereal for lunch?
>
> Which is more likely, that a boy will build a model airplane or that a girl will build a model airplane?

Some things are more likely to happen than not. Think of answers to these questions:

> In Phoenix, Arizona, in July, is it more likely than not that the sun will be shining at noon?
>
> Is it more likely than not that you can find the sum of 324 and 465 ?

Is it more likely than not that your neighbor has a TV set?

Some things are certain and some things are impossible. Which are these?

A man can live without any liquid for three months.
I will use my brain some time this week.
My dog can write his first and last name in Russian.
All new cars this year will use water for fuel.
Tomorrow, today will be yesterday.

Our ideas about chance might be classified Certain, Uncertain, or Impossible. In front of the following sentences, write C, U, or I for Certain, Uncertain, or Impossible.

_____ 1. Sun will set in the east.

_____ 2. A river flows downhill.

_____ 3. We will see the sun today.

_____ 4. Sun will rise

_____ 5. A river flows uphill.

_____ 6. A river is deeper today than yesterday.

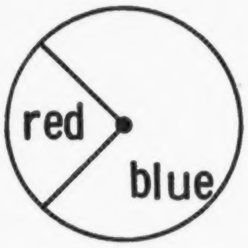
_____ 7. I will sleep 8 hours on Monday.

_____ 8. I will sleep sometime this week.

_____ 9. I will not sleep at all this week.

Exercises - Lesson 1.

Use these pictures for Exercises 1 through 3.



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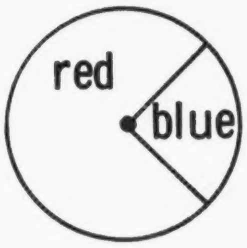

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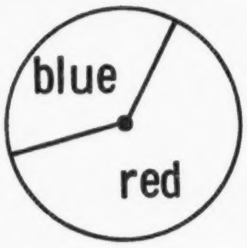

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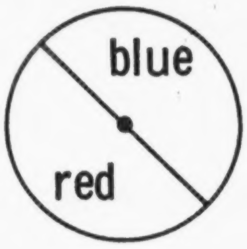

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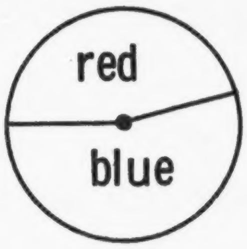

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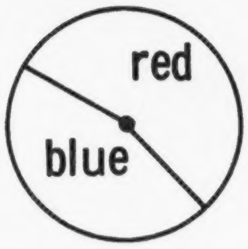


F

1. Circle the letter of the spinner whose pointer is more likely to stop on red than blue.

   (a) A or B ?
   (b) C or D ?
   (c) E or F ?
   (d) B or C ?
   (e) D or E ?
   (f) C or F ?

2. Study spinner D and answer these questions.

   (a) Could you get 100 reds in 100 spins on this spinner? ____________

   (b) Are you likely to get 100 reds in 100 spins on this spinner? ____________

   (c) About how many reds do you expect from 100 spins? ____________

3. Suppose a pirate captain said to you, "I will give you just one chance on a spinner. If the pointer stops on blue, into the sea you go. If it stops on red, you may go free."

   (a) If the captain let you choose one of these six spinners, which would you choose for your chance? ____________

   (b) If the captain allowed you to make the spinner, how would you color the dial? ____________

   (c) If the captain were very angry, how do you think he would color the dial? ____________

Things To Do At Home - Lesson 1.

1. Look for stories in the newspaper that use some of these words.

   probable
   chance
   likely
   probability
   equal chance
   unlikely

   Bring them to share with your class.

DISCUSSION OF EXERCISE

(Lesson 1)

The exercise for lesson 1 was done as homework and was collected the following day, Thursday. The exercise was returned to the students on Monday.

Many students scored below 90 per cent on the exercise, having particular difficulty in classifying instances of certainty, uncertainty, impossibility. Only 9s/25s were classified as masters initially (criterion--two or fewer mistakes). The rest were told to correct their mistakes and show them to the observer. The questions on page three were discussed in class in detail.

The exercise concerning the classification of spinners on page four did not cause any problem except for those who reversed the colors.

At the bottom of page four, many children did not answer 2(a) and (b) by the response of "yes" or "no." They used certain, uncertain, and impossible. For question 2(e) the common response was "uncertain."

On page five some children thought that it was necessary to have both colors on the spinners. Thus, they would write an answer or draw a picture to show a spinner with just a little of one color and a great deal of the other color.

As far as the Things to do at Home is concerned, it was not stressed. One girl brought in an article concerning a weather report the following day. The teacher did not refer to the article in class.

Lesson 2

EXPERIMENTATION

Objectives:

The child should be able to:

1. Identify an experiment (1-D).
2. Perform the given experiment.
3. Collect the data.
4. Record the data.

Prerequisite Behaviors:

Counting

Materials to be used:

1. Spinners (1/2R, 1/2W); (3/4W, 1/4R).
2. Marbles (1 R, 1 B, 1 Y); (3W, 1 R); Quarters; Die.
3. Team Cards, Team Data Sheets, Summary Report Sheets, Directions for the Experiments.

New Vocabulary:

data

Method of Presentation:

1. Review lesson and vocabulary of yesterday by taking a die and asking the class the following questions.
    a. What are the possible outcomes when I throw the die and observe the face that turns up.

b. How many outcomes are even numbers? Odd numbers? What can we say about the chances of getting an even number compared to the chances of getting an odd number?

c. In terms of certain, uncertain or possible, and impossible, how would you classify the following events?

   1. the chances of getting an odd number.
   2. the chances of getting the number "5".
   3. the chances of getting a number greater than 7 (less than 7).

d. In terms of more likely, less likely or equally likely, classify the following events. The chances of getting:

   1. an even number compared to an odd number.
   2. the number of "6" compared to the number "2".
   3. a number greater than "2" compared to a number less than "2"

2. Introduce today's lesson by saying, "Today we are going to carry out the experiments we talked about yesterday. We are going to collect, organize and eventually make bar graphs of the data from the experiment. By doing this, we will learn more about chance events and the ideas we talked about yesterday."

3. "When you are through with an experiment, go to the ledge in the front of the room and find the Summary Report Sheet for that activity. Your data should be recorded opposite your team number. Please put your initials beside your team's number. Then you should find an experiment that is not being used. Go and do that one. I expect that you should be able to do about

five of the six experiments in the time remaining. Are there any questions before we begin?"

4. The teacher at this point should direct one team at a time to an experiment.

> Comment:
>
> The children were allowed to choose their own experiment.

5. At the end of the session the teacher is to collect each data sheet.

Detailed Directions for Experiments:

"The directions for performing each experiment are beside the models. Let's take a quick look at each experiment." (Briefly explain what is to be done for each experiment, e.g., First Activity: You spin this spinner 40 times and record the number of blue and the number of red that occur.)

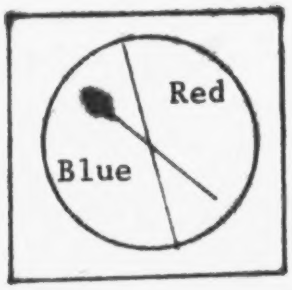
"When we are performing these experiments, we must be careful to do the experiments fairly. Can anyone think of how we could do the experiments unfairly?" (They should give responses such as: spinner--stopping it; marbles--looking at or not mixing up; die--not shaking or allowing to bounce; same for coin, etc.) "One reason for needing fairness is because the data from the different teams performing an experiment will be added together."

"Now to do these experiments you and your neighbor will form a team of two." (Assign a number to each team on a 3 x 5 card with the names on it and tell the team members to remember its number, e.g., Team 1, Team 2, etc.) "I will assign which experiment your are to do. First read the directions for that experiment carefully. One person is to act as the recorder on the sheet that I will give each team (hold up), while the other performs the experiment. Each experiment is to be done 40 times. The two people may change their duties after the experiment has been performed say, 20 times."

"Before you actually perform the experiment you are to record your guess as to the number of occurrences you think you will get for each possible outcome. If you people have different guesses, record both numbers."

Lesson 2

Activity 1: Spinning a spinner with dial 1/2 blue and 1/2 red.



Directions:

The pointer of the spinner is to be spun and a record made of whether it stops on red or blue. (If it stops exactly on a line between red and blue, make no record, but spin again.)

One member of the committee will serve as recorder. The other members of the committee will take turns spinning the pointer until a total of 40 spins have been recorded on the record sheet headed "Activity 1".

Here is a sample record of 20 spins to show how the count should be kept:

20 trials. Guess the number of red __10__ and the number of blue __10__ you think you will get.

Number of Red		Number of Blue		Total Number of Spins
𝍸 |||	8	𝍸 𝍸 ||	12	20

Write down the number of red and blue you guess you will get in 40 spins before you carry out the experiment. Record the results of 40 spins. Check the results to make sure they add up to 40 and then go on to the next experiment.

Comment:

An Activity Sheet was placed beside each experiment.

Lesson 2

LIBERTY
IN GOD WE TRUST
1963

heads

UNITED STATES OF AMERICA
QUARTER DOLLAR

tails

Activity 2: Tossing a coin.

Directions:

The coin is to be tossed and a record made of whether it falls with heads or with tails showing.

One member of the committee will serve as recorder. The other members of the committee will take turns tossing the coin, until a total of 40 tosses have been recorded on the record sheet headed "Activity 2".

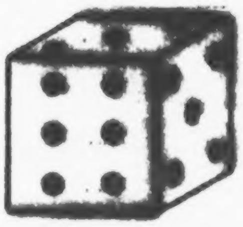
Here is a sample record of 20 tosses to show how the count should be kept:

20 trials. Guess the number of heads 10 and the number of tails 10 you think you will get.

Number of Heads		Number of Tails		Total Number of Tosses
𝍸 ||||	9	𝍸 𝍸 |	11	20

Write down the number of heads and tails you guess you will get before you carry out the experiment. Record the results of the 40 tosses. Check the results to make sure they add up to 40 and then go on to the next experiment.

Lesson 2



Activity 3: Tossing a die and noting whether the number of dots on the top face is even or odd.

Directions:

The die is to be tossed and a record made of whether the number of dots on the top face is an even number (2,4, or 6) or an odd number (1,3, or 5).

One member of the committee will serve as a recorder. The other members of the committee will take turns tossing the die, until a total of 40 tosses have been recorded on the record sheet headed "Activity 3".

Here is a sample record of 20 tosses to show how the count should be kept:

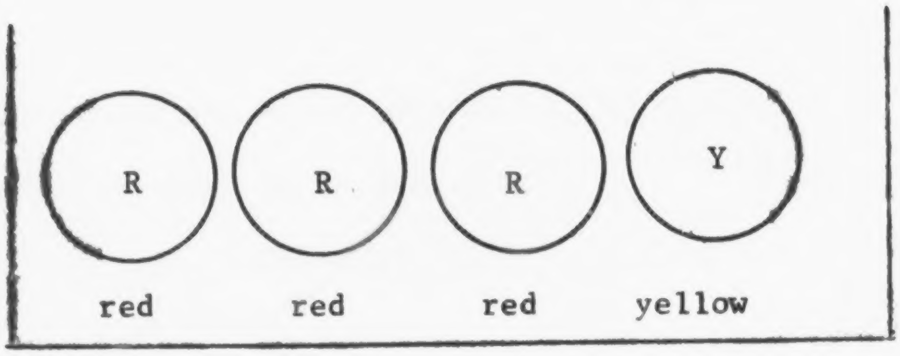
20 trials. Guess the number of even numbers 10 and the number of odd numbers 10 you think you will get.

Number of times even		Number of times odd		Total Number of Tosses
𝍸 ||	7	𝍸 𝍸 |||	13	20

Write down the number of even numbers and the number of odd numbers you guess you will get before you carry out the experiment. Record the results of the 40 tosses. Check the results to make sure they add up to 40 and then go on to the next experiment.

Lesson 2

Activity 4: Choosing a marble from a box containing one yellow and three reds.



Directions:

From a box containing three red marbles and one yellow marble, none visible, a single marble is chosen and its color recorded.

One member of the committee will serve as recorder. The other member(s) of the committee will take turns choosing marbles until a total of 40 choices have been recorded on the record sheet headed "Activity 4".

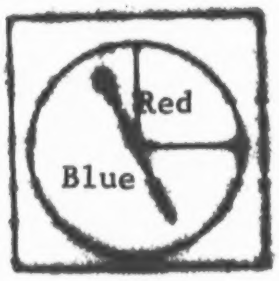
Here is a sample record of 20 choices to show how the count should be kept:

20 trials. Guess the number of Red 15 and the number of Yellow 5 you think you will get.

Number of Red		Number of Yellow		Total Number of Choices
𝍸 𝍸 𝍸 𝍷	16	𝍷𝍷𝍷𝍷	4	20

Be sure to write down the number of red and blue you guess you will get in 40 choices before you carry out the experiment. When all 40 choices have been recorded and checked by the committee to make sure they add up to 40, go on to the next experiment.

Lesson 2



Activity 5: Spinning a spinner with dial 1/4 red and 3/4 blue.

Directions:

The pointer of the spinner is to be spun and a record made of whether it stops on red or blue. (If it stops exactly on a line between red and blue, make no record, but spin again.)

One member of the committee will serve as a recorder. The other members of the committee will take turns spinning the pointer, until a total of 40 spins have been recorded on the record sheet headed "Activity 5".

Here is a sample record of 20 spins to show how the count should be kept:

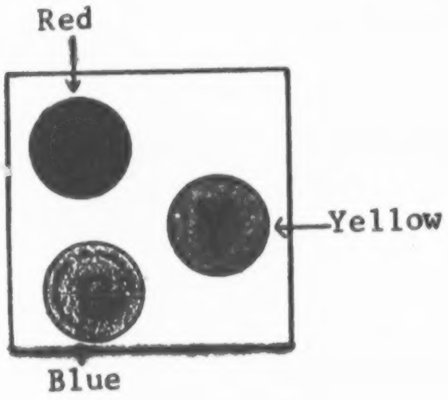
20 trials. Guess the number of Red 10 and the number of Blue 10 you think you will get.

Number of Red		Number of Blue		Total Number of Spins
𝍸 |||	8	𝍸 𝍸 ||	12	20

Be sure to write down the number of red and blue you guess you will get in 40 choices before you carry out the experiment. When all 40 spins have been recorded and checked by the committee to make sure they add up to 40, go on to the next experiment.

Lesson 2

Activity 6: Choosing a marble from a box containing one red, one blue, and one yellow.



Directions:

From a box containing one red marble, one blue marble, and one yellow marble, without looking, a single marble is chosen and its color recorded.

One member of the committee will serve as recorder. The other members of the committee will take turns choosing marbles until a total of 40 choices have been recorded on the record sheet headed "Activity 6".

Here is a sample record of 20 choices to show how the count should be kept:

20 items. Guess the number of Red 7, the number of Blue 6, and the number of Yellow 7 you think you will get.

Number of Red		Number of Blue		Number of Yellow		Total Number of Spins
𝍸	5	𝍸 |||	8	𝍸 ||	7	20

Be sure to write down the number of red, blue, and yellow you expect to get in 40 choices before you carry out the experiment. When all 40 spins have been recorded and checked by the committee to make sure they add up to 40, go on to the next experiment.

Lesson 2

Names of Committee Members ____

Team ________

Activity 1. 40 trials. Guess the number of red ________ and the number of blue ________ you think you'll get.

	Number of Red		Number of Blue		Total Number of Spins
Spinning a spinner (1/2 R, 1/2 B)					

Activity 2. 40 trials. Guess the number of heads ________ and the number of tails ________ you think you'll get.

	Number of Heads		Number of Tails		Total Number
Tossing a coin					

Activity 3. 40 trials. Guess the number of even numbers ________ and the number of odd numbers ________ you think you'll get.

	Number of even numbers		Number of odd numbers		Total Number of Spins
Tossing a die					

Comment:

A recording sheet was given to each team.

Activity 4. 40 trials. Guess the number of red ________ and the number of yellow ________ you think you'll get.

	Number of Red		Number of Yellow		Total Number
Choosing a marble (1Y, 3R)					

Activity 5. 40 trials. Guess the number of red _______ and the number of blue ______ you think you'll get.

	Number of Red		Number of Blue		
Spinning a spinner (1/4 R, 3/4 B)					

Activity 6. 40 trials. Guess the number of red ______, the number of blue ______, and the number of yellow ______ you think you'll get.

	Number of Red		Number of Blue		Number of Yellow		Total Number
Choosing a marble (1R, 1B, 1Y)							

Summary Report Sheet

Activity ______________________________

Teams	Number of ___	Number of ___	Number of ___	Total Number
Team 1				
Team 2				
Team 3				
Team 4				
Team 5				
Team 6				
Team 7				
Team 8				
Team 9				
Team 10				
Team 11				
Team 12				
Totals				

Comment:

Summary Report Sheets were placed beside each experiment.

Summary Report Sheet

Activity ________________________________

Teams	Initials	Number of ___	Number of ___	Total Number
Team 1				
Team 2				
Team 3				
Team 4				
Team 5				
Team 6				
Team 7				
Team 8				
Team 9				
Team 10				
Team 11				
Team 12				
Totals				

DISCUSSION OF LESSON 2

Thursday (3/6)

As the lesson was about to begin a student asked a question concerning the problem of whether a river can flow uphill. The question was discussed and the conclusion reached was that it depended on one's interpretation of the problem and, therefore, either Certain or Uncertain was acceptable. The homework was collected so that the assignment could be corrected.

This discussion and the review of Lesson 1 using the die took approximately 15 minutes.

The chart of the goals was employed by the teacher to show students the specific goals of today's lesson and their relation to the pre-instruction on graphs and Lessons 1 and 3. The teacher went over each experiment and explained how to record the data on the sheets which had all the experiments listed and where to do the tallying. She also showed them on the board how to tally. She stressed that they were to carry out the experiment fairly. Their names were already on the recording sheets and she explained that one person was to do the experiment while the other child recroded. Two people would be working together. When they were half way through one experiment, they could switch around and the one who was recording could now do the experiment and the one who had been doing the experiment could record. When they were finished with an experiment, they were to total the results and record it on the Summmary Report Sheet.

The teacher then passed out recording sheets and the children went to work. The teacher and the observer circulated among the children and answered any questions they had. The most common comment made was to remind them to record their data on the Summary Report Sheet and how to do this recording.

The children responded to the experiments quite well with many asking if they could do more of these activities tomorrow. This in part could be explained by a team's desire to complete all of the activities. The teams completed an average of four of the six activities in the 30 minutes available to them.

The recording sheet asked the children to guess the number of times each outcome would be gotten in 40 trials. Most did not record the expected value; rather they recorded numbers like 21, 19, etc.

The children seemed to be very conscious of trying to perform the experiments fairly, even though the results did not match what they had initially guessed. For example, when picking from the box, many would hold the box above their eye level. When one team member got two marbles by mistake, he did it over; or when one child stopped a spinner with his finger, his partner told him he couldn't count the result and that it would have to be done over.

The instructions and forming of committees seemed excessively complicated, although the students were able to follow the directions quite well. The use of 11 teams of two students and 1 team of three students would seem to be too much for one teacher. There would probably be too much confusion and too much work involved in getting all of the models ready for one period of instruction. Perhaps teams of three or four members with fewer models would be just as beneficial.

Lesson 3

GRAPHING OF RESULTS

Objectives:

The child should be able to:

1. Construct a bar graph of data from an experiment.
2. Identify the randomness of the frequencies of the results from the different committees for an experiment.

Prerequisite Behavior:

Ability to construct and interpret bar graphs--given the data.

Materials to be used:

graph paper, ruler, results from experiments, practice sheet, poster paper.

New Vocabulary:

None.

Method of Presentation:

1. On an overhead, present data from an experiment. (See the following sheet--experiment: Drawing one marble from a box containing one red and one blue marble.) Review how to construct a bar graph (teams x frequency) of the data by actually constructing one on the overhead.

Comment:

Done on blackboard.

Help students to interpret the results from the bar graph by asking the following questions:

a. What can you say about results of this experiment?

Comment:

No response

If this elicits no response or to aid their thinking, ask them questions such as:

1. Which committee got the most red? The least red? How many of each?
2. Which committee got the most blue? The least blue?
3. Can one predict the number of red one will get in 40 trials?
4. Can one tell where most of the number of red fall (between what two numbers?)
5. Let's look at the total results. In 200 trials, how many red do you expect you should get? Let's add them up and find out how many were actually gotten.

The cumulative graph of the data should be made on the overhead at this point.

"Let us now form committees to look at the data from yesterday's experiments and make bar graphs of this data. These graphs will help us to interpret the results. We would like you to form committees of four by combining team 1 and team 2, team 3 and team 4, . . . team 11 and team 12. Each committee will be given a Summary Report Sheet from one experiment. I would like you to take the large poster paper that I will give you and have you make two graphs of the data you will have.

1. The results from an experiment are uncertain. Do this by asking the class whether we can predict what a team will get in 40 trials in an experiment. (Should say no--if they don't, just hold up graph of team results and repeat question.)
2. Two events may or may not be equally likely. Compare results from coin and dice in contrast say to spinner (3/4 B, 1/4 R) and ask them to state why they think the results are somewhat the same in the former case and quite different in contrast to the latter.
3. We can never be certain of the exact outcome of chance events. Ask: "Can we predict for certain what will be the particular outcome in doing any of the experiments one time?
4. There should be a pattern in chance events over a large number of trials. This pattern may enable us to forecase the likely bounds on the frequency of the occurences of the event.

Note: Pass out practice sheets.

One graph is to show the results of each team and a second graph is to show the results when the team results are added together, just like we showed you on the overhead. Each team can work on one graph. Then the four of you should look at your graphs and try and interpret them like we just did. When you finish, bring your work to the teacher.

The committee is to present their findings first and then, as needed, the teacher should ask questions as in the above sample problem to aid in the students' interpretation of the graphs.

Results from equivalent and non-equivalent models should be compared by displaying the posters and noting similarities and dissimilarities in the graphs.

The children should note or questions should be directed to aid the children to observe the large differences in frequencies for the unequally likely experiments (3 red marbles, 1 yellow marble; spinner (3/4 B, 1/4 R)) in contract to the experiments which are equally likely (die, coin, spinner (1/2 R, 1/2 B), 3 differently colored marbles). The teacher should ask students to compare graphs of the three equivalent experiments (die--even and odd, coin, spinner--1/2 R, 1/2B). There ought to be somewhat similar results when the cumulative graphs of each of the models are compared. From the graphs the students should be encouraged to state the likely frequency bounds of an outcome for a given number of trials. Ask this question pointing to a graph: Can you give me an upper number and a lower number of _____ you feel your results from an experiment done 40 times would be between most of the time? If I tossed a coin 1000 times, what would be a "good" upper number and lower number which the number of heads I get would be between? The teacher should encourage the students to verbalize their findings. Among these should be:

QUESTIONS TO USE WHEN TALKING ABOUT EXPERIMENTS

For Spinner 1/2 R and 1/2 B Ask:

Do these results agree with what you expect?

Did you expect R and B to be equally likely or is one color more likely than another?

Why do you think R turned up much more often than B?

Do you think a team would get the same results if they did it again?

Is it possible to spin 40 times and get 40 R? Is this likely to happen?

Coin:

Do these results agree with your expectations?

Should the results have come out about the same? Did they?

Could you get 20 heads in a row? Is this likely?

If a team were to do this again, could you predict what would happen?

Could you tell me the upper number and lower number of heads you feel your team would get most of the time for 40 tosses?

Look at the graph to help you. Can you give me the two numbers from the graph which are closest together that most of the results will probably lie between?

Do the same with a die.

Ask students to compare these three models (experiments with spinner (1/2 R, 1/2 B), quarter, and die (even, odd)). What can be said about them? (They all have two equally likely outcomes.) Did the results show this approximately? For which models?

How about the results of the spinner (3/4 B, 1/4 R)? How does it compare to these? (Two outcomes but not equally likely) Was there any activity that is like this model? (Yes - 3 R, 1 B marble.)

Comment:

This sheet was passed out to students to help them in their interpretation of their graphs.

To help you interpret your graphs think about the following questions:

1. Which team had the most ___________?
2. Which team had the least ____________?
3. From the graph what looks like the average number of _________ that was gotten by a team? Does this agree with what you expected you would get? If not, why not?
4. Can you predict the number of ____________ you would get in 40 trials?
5. Look at the graph where the total results are added together. How does the number of ___________ compare to the number of ____________? In this what you would expect?

INTRODUCTORY DISCUSSION OF LESSON 3

Lesson 3 was scheduled to take two days. However, the graphing, presentation and interpretation of the graphs and the exercises for Lesson 3 took Friday, Monday, Tuesday and twelve minutes on Wednesday (March 7, 10, 11, and 12).

Friday and Monday were spent in constructing the graphs. Part of the period on Tuesday and Wednesday was used to present and interpret the results of the graphs. The other part of the Tuesday session was used to complete the exercise that is to accompany Lesson 3. The rest of Wednesday's session was used to introduce Lesson 4.

Discussion of Lesson 3--First Day, Friday, March 7

The bar graphs that the children made Wednesday of the number of children in a family were passed back at the beginning of the lesson. The common mistakes of not labeling axes, misplacement of numbers on the vertical scale, and sloppiness in drawing the graph were mentioned by the teacher. The class was told to correct their mistakes and show them to the experimenter in order to get a M (Master's) on the assignment. Some children copied their graphs over because mistakes couldn't be erased.

The teacher then placed data on the board from an experiment of drawing one marble from a box containing one red and one blue marble. Following a discussion on how to scale the vertical and horizontal axes, the graph of the individual trials (50 in a trial) was made. The question was raised by the teacher as to how one could represent the number of red and blue from one trial with one bar. The teacher first discussed how to represent both the red and blue with one bar before she drew the bars. The children didn't suggest putting a red bar in and extending it to represent the blue. The teacher showed how to do this with team 1's results. Several children still looked puzzled. She then said that "when you add each team's red and blue results together, what do you get each time? (Children said "50.") So you see the red and blue bars will always equal 50. If the red is small, the blue will be large. If the blue is small, what will the red bar look like? (Children said "Large.") When you make your graphs, the first bar can be the red or blue. If you're working with a coin, it will be heads or tails and with the die it will be either odd or even." Thus, it was suggested by the teacher that all the graphs of experiments having outcomes would use a two-colored single bar to represent a team's results. The teacher commented that if you have the data from the spinner divided 1/3--1/3--1/3, you will need three bars.

The class had no trouble answering the questions that were to aid their interpretation of the graphs.

The cumulative graph of the data was then made by the teacher on the board. Unfortunately the words "cumulative graph" were used without defining what these words meant. The graphs were drawn quickly on the board and were not too clear or accurate.

Two teams were assigned to form a committee to take a Summary Data Sheet of the data for an experiment that was done in Lesson 2 and make two bar graphs of the results. The committee assignments were decided before class. Two people on the committee were to work on the team results and the other two were to work on the total results.

The first part of this lesson lasted about 25 minutes. The children had 25 minutes to begin their graphs. The posters used for graphing by the team had the axes premarked.

Discussion of Lesson 3--Second Day, Monday, March 10

The lesson started by asking the children: "How many people knew which activity they were graphing?" Some knew which activity they had graphed and some did not. One committee entered into a debate whether they had done the experiment of tossing a die or tossing a coin.

Each committee was given a sheet with questions they were to answer to help them interpret the data. The teacher went over the questions with the class to help them understand what was expected of them.

This part of the lesson lasted six minutes. The next 30 minutes were spent in completing the graphs. Snopake and gum labels were used to patch the mistakes that had been made.

The teacher and the observer each worked with three committees. The children needed reminding to label scales and to write the name of the graphs on the poster.

Most committees were very efficient. However, it seemed that the two committees that were made up of all boys were not as efficient as the others. The four committees consisting of all girls or two boys and two girls worked well together.

The last nine minutes of the period were spent going over the homework from Lesson 1. (This was handed in on Thursday.) The questions were discussed, particularly the questions where the children had to identify instances of certainty, uncertainty, or impossibility. Since the observer had corrected the questions rather than the teacher, she was not familiar with the interpretations. This led to some confusion as to which answer was acceptable. The author then stated the accepted answers and told why they were accepted. The nonmasters were told to show their corrections to the experimenter.

After discussing the lesson with the experimenter, the teacher volunteered to correct the exercises in order to be more familiar with them.

Some committees had not completed their graphs. They were told to do this during their free time before tomorrow's class. The committees were told that they would be presenting the results of their graphs Wednesday and should use their question sheets to help them in interpreting their graphs.

Discussion of Lesson 3--Third Day, Tuesday, March 11

The lesson began with Committee 1 presenting its graph of the experiment of spinning the 1/2 R--1/2 B spinner. The questions on the sheet passed out previously were discussed, with the committee asking the class the questions. The class thought that, on the average, the results should lie between 20 and 26 red. The class seemed to agree that if they did the experiment again that it was uncertain as to the number they would get in 40 spins. Also, they seemed to feel (at least some expressed this verbally without the others disagreeing) that the fact that there was more red than blue did not mean the red was more likely than blue and that just the opposite results could happen the next time.

Committees 2 and 3 presented their results concerning tossing a coin and a die respectively. Each committee went to the front of the room and displayed their two graphs. Sometimes the committee would ask the class the questions. At other times, the teacher would have to ask the questions. All the answers came from the class rather than the teacher.

The three graphs were displayed together and compared briefly by the teacher. The fact that the graphs represented experiments that had two equally likely outcomes was brought out by the teacher asking the students questions. The expected value or the number one would expect to get was pointed out.

The fact that the results of the cumulative graphers for the spinner (1/2 R, 1/2 B) showed quite a difference in the two bars compared to the slight differences in the coin and die graphs was brought out by the teacher. These presentations and the comparison by the teacher lasted 30 minutes.

The quiz on Lesson 3 took the last 20 minutes of the class.

A homework assignment concerning interpreting the results of bar graphs was passed out at the end of the period.

Discussion of Lesson 3--Fourth Day, Wednesday, March 12

The last three graphs were presented Wednesday. The graphs were reviewed very quickly with the teacher asking a few questions on each graph.

The graphs of tossing the coin and choosing a marble from a box (3 R, 1 Y) were compared. A girl explained the differences were because "heads" has 1/2 a chance while the yellow marble has 1/4 chance. Therefore, one would expect more heads than yellow.

The above activities lasted 12 minutes.

EXERCISE 3.1*

Exercises - Lesson 3.

Here are ten statements about chance events. If you think a statement is true, put a T in the blank after the statement. If you think the statement is not true, put an F in the blank.

1. If a tossed coin does not stand on its edge, it is certain to be either heads or tails. ______________
2. If we toss a coin once, we are as likely to get a head as a tail. ______________
3. If we toss a coin 100 times, it may be heads 0 times or 100 times or anything in between. ______________
4. If we toss a coin 1000 times, it is very unlikely that we will get 900 tails. ______________
5. Whether we get heads or tails when we toss a coin is a matter of chance. ______________
6. You might toss a coin 1000 times without getting a single head. ______________
7. If a box contains two blue marbles and one red one, and you pick one marble without looking, the chances are 2 out of 3 that it will be blue. ______________
8. In Exercise 7, you have one chance in three of picking a red marble. ______________

---

*From SMSG's Probability for Intermediate Grades, pp. 17, 18, 20.

9. In Exercise 7, your chances of picking a green marble are zero. ______________

10. Joe is eight years old. It is more likely that he is four feet tall than ten feet tall. ______________

Read the following statement carefully and then answer Questions 11 to 15.

> A spinner has a dial which is one-fourth white and three-fourths red.

11. If you spin the pointer 10 times, are you likely to get the same number of reds as whites? ______________

12. Are you likely to get more whites than reds? ______________

13. If the chances of getting red are 3 out of 4, what are the chances of getting white? ______________

14. Can you be certain of getting at least one red in 10 spins? ______________

15. Is it very likely that you will get no reds in 10 spins? ______________

Read the following statement carefully and then answer Questions 16 through 20.

James has three green marbles and
two blue marbles in his pocket.

16. How many marbles must he remove to be sure of getting a blue marble? ____________

17. How many marbles must he remove to be sure of getting both the blue ones? ____________

18. How many marbles must be removed to be sure of getting both colors? ____________

19. How many marbles must be removed to be sure of getting a green one? ____________

20. If James removes one marble, there are three chances out of ___?___ that it will be a green one. ____________

Think about some things that are certain to happen. Think about some things that might happen, and about other things that just can't happen. Then answer questions 21, 22, and 23.

21. List three things that you know are certain to happen.

a. ____

b. ____

c. ____

22. List three things that may or may not happen.

a. ____

b. ____

c. ____

23. List three things that cannot happen.

a. ____

b. ____

c. ____

DISCUSSION OF EXERCISE 3.1 (LESSON 3)

The exercise was done as an independent assignment in class. The exercises were collected and graded. The mean was 19.21 out of 24 responses, and the variance was 11.00.

Of the 24 people taking the quiz (one absent), only 8/24 were masters (criterion: number of misses less than or equal to 3). The item difficulty for each item in ratio form was:

1. 24/24
2. 22/24
3. 21/24
*4. 11/24
5. 24/24
6. 19/24
7. 22/24
8. 22/24
9. 23/24

10. 23/24
11. 22/24
12. 19/24
13. 18/24
*14. 15/24
15. 20/24
*21. $\left.\begin{array}{l}a\\b\\c\end{array}\right\}48/72$

22. $\left.\begin{array}{l}a\\b\\c\end{array}\right\}69/72$
*23. $\left.\begin{array}{l}a\\b\\c\end{array}\right\}39/72$

Items 4, 14, 21, and 23 are of special interest because of the number of incorrect responses. Items 4 and 14 were probably missed because of the misunderstanding of verbal phrases, "it is very unlikely..." (item 4), and "at least one red..." (item 14). In item 21, listing three instances of certainty was difficult for 14 persons. (They had 1 or more wrong.) Of the 14 children, 7 had one wrong instance of certainty, the other 7 had two or more wrong instances.

The common mistake was to cite examples of almost certainty as certainty. Examples of this are:

"I will eat something today."

"I will have gym today."

"I am going to sleep tonight."

Impossibility was not as difficult as the number of wrong responses reflect. Again as in 4, the negative statement, "list three things that cannot happen," seemed to confuse them. The common mistake (30/33 errors) was to include "not" in an otherwise impossible instance. Correct responses included these:

"My mother won't have a baby tomorrow." (She isn't going to have a baby.")

"The sky will not fall."

"I can't be 15 in November."

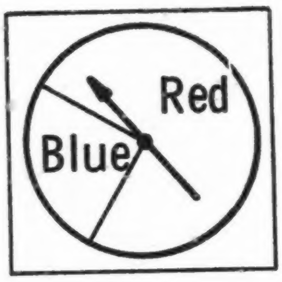
"The earth will not fall on the moon."

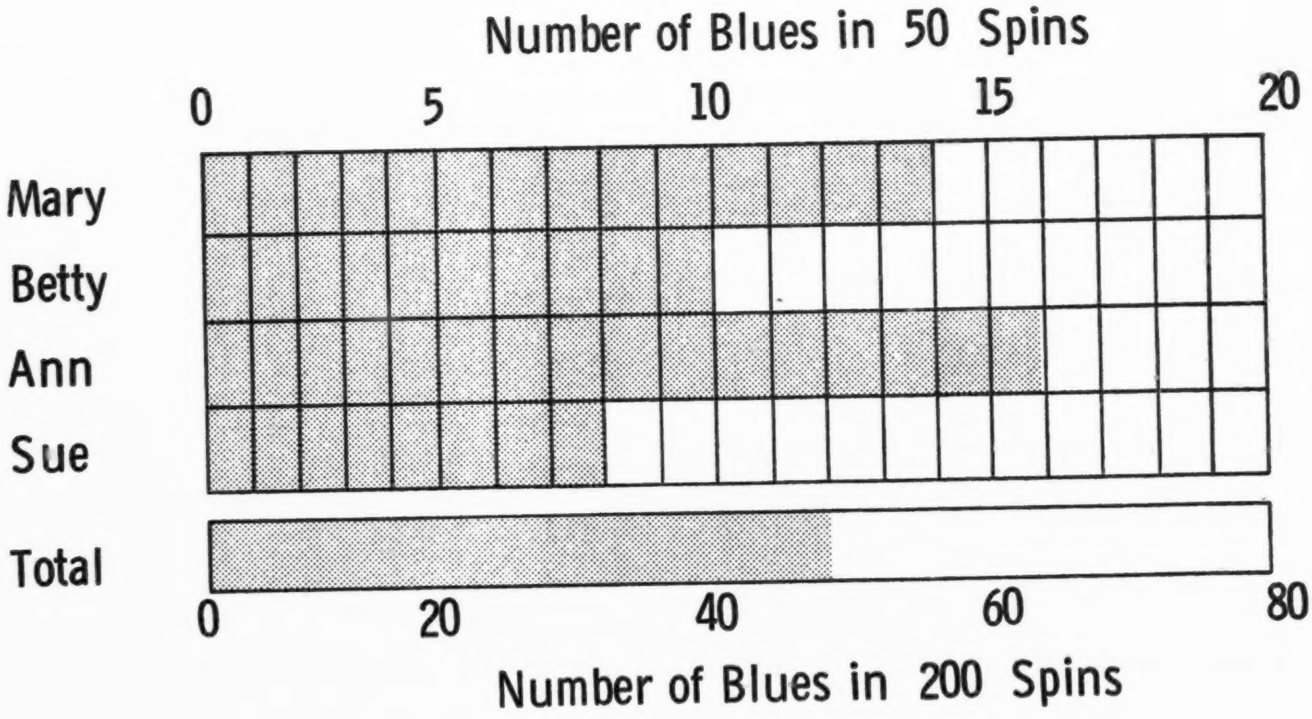
"No one can turn into an insect."

The papers were given back to the students after prescriptions for correcting them were written on the papers. Most of the students had no problems in correcting them once the fallacy of their errors was discussed in class. Eventually all but one succeeded in correcting their mistakes. Children scoring initially below 75 per cent were subjects 15, 22, 24, and 26.

EXERCISE 3.2*

3. Below is a bar graph of the results some girls found in using the spinner at the right. You can read it in the same way you do other bar graphs. Look at it carefully, and you will see how to do this. Use it to answer a through h.





a. Who had the smallest number of blues in 50 spins? ________

b. Who had the largest number of blues in 50 spins? ________

c. How many reds did Betty get in 50 spins? ________

d. Which of these fractions tells about how much of the dial is blue?

$\frac{1}{2}$ $\frac{1}{4}$ $\frac{3}{4}$ ____________

e. Did any girl get 25 or more blues? ________

f. How many times in all was the spinner spun by the girls? ________

g. How many of these spins ended on blue? ________ Is this about the number of blues you would expect on 200 spins? ________

h. Would you rather guess the number of blues on 20 spins or on 200 spins? __________ spins.

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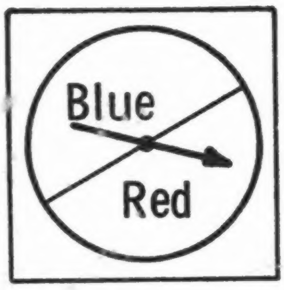
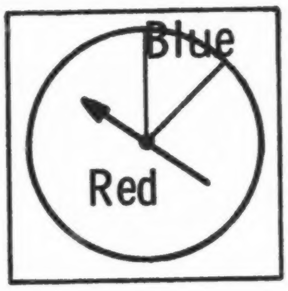
*Taken from SMSG's Probability for Intermediate Grades, pp. 35-37.

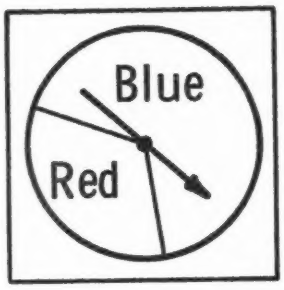
4. Here are some more spinners and graphs.



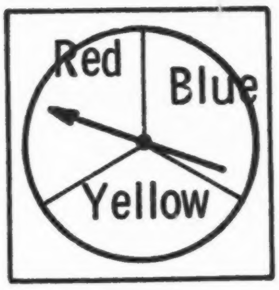
1



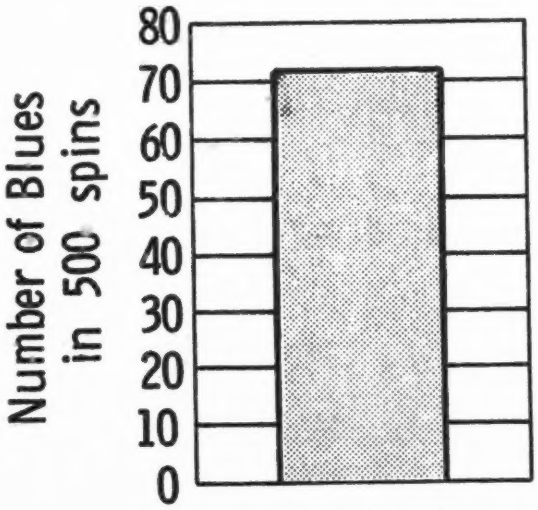
2



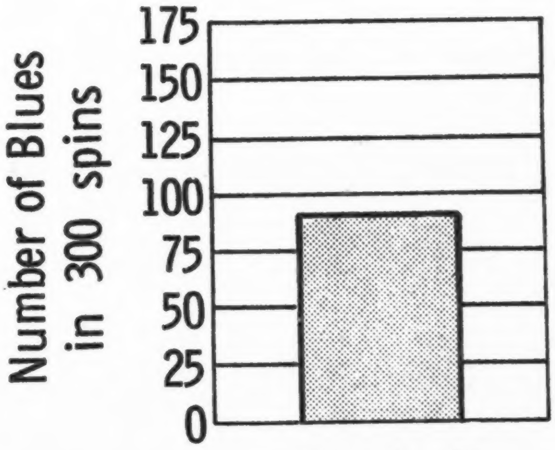
3


Red
Blue
Yellow

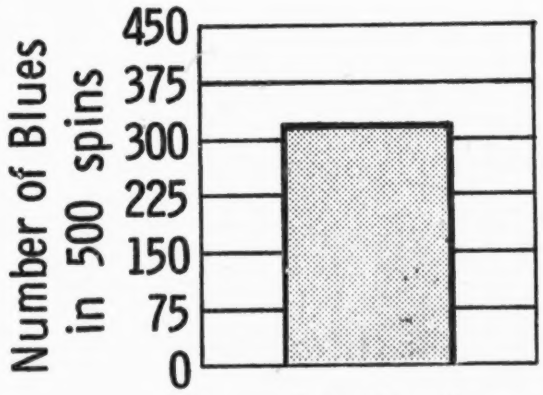
4



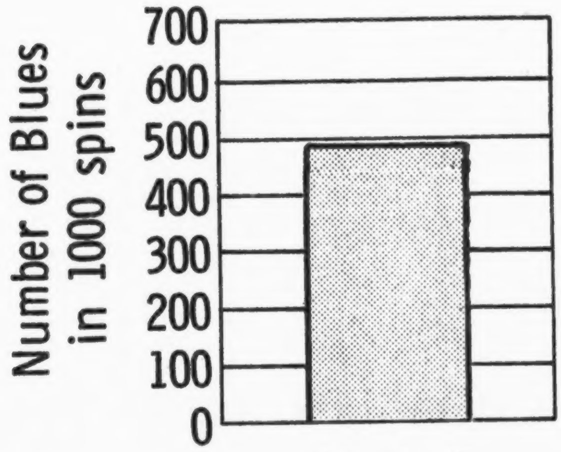
Graph A



Graph B



Graph C



Graph D

a. Graph A was probably made by using data from spinner ________.

b. Graph B was probably made by using data from spinner ________.

Comment:

This exercise was done as a homework exercise.

Due to a lack of time it was never handed back.

4. (Continued)

c. Graph C was probably made by using data from spinner ________.

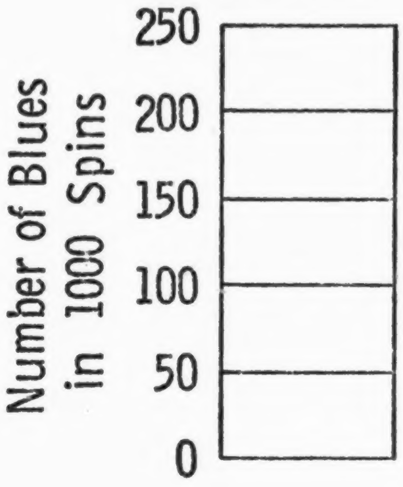
d. Graph D was probably made by using data from spinner ________.

e. Which spinner would you choose if you wanted to be most likely of getting blue? __________

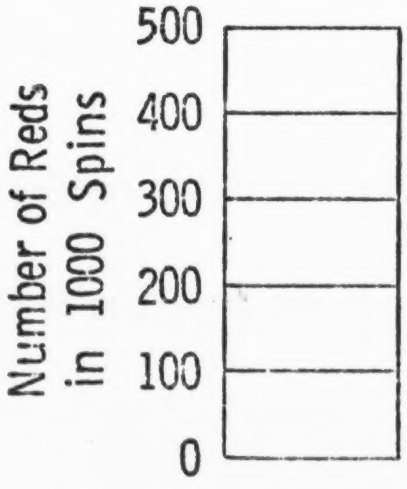
f. On which spinner is red more likely than blue? ________

g. One of the spinners is spun 10,000 times. Blue was the result 3,300 times. Which spinner would you expect was used? ________

h. Spinner 2 is spun 1,000 times. Draw a bar graph to show the number of blues you would expect.



i. Spinner 3 is spun 1,000 times. Draw a bar graph to show the number of reds you would expect.



COMMENT ON LESSON 3

The use of poster paper may not be the best way to graph data. The children were very slow in making the graphs--particularly in coloring them.

Because of the length of time and effort involved in making the graphs, some children forgot what they were graphing. For example, two girls finished their cumulative graphs of the marble experiment (3 R, 1 Y) and presented the graph to the teacher. However, they had not labeled the graph so that one could tell which experiment it came from. When this was pointed out to them, they drew a spinner (1/2 R, 1/2Y) on their poster--although their poster showed 80 Y and 240 R in 320 trials.

If possible, it probably would have been more efficient to have had children use overlays and make their graphs within a very structured format.

Lesson 4

PROBABILITY OF EQUALLY LIKELY OUTCOMES AND EVENTS

(One-Dimensional Finite Sample Space)

Objectives: The child should be able to:

1. Specify the probability of an equally likely outcome.
2. Specify the probability of an equally likely event as the number of favorable outcomes/number of possible outcomes.
3. Specify the probability of the sure event.
4. Specify the probability of the null event.
5. Specify the probability statement by use of the appropriate notation (e.g. P(R) = 1/2 means the probability of drawing a red marble is 1/2.)
6. Identify chance and probability as being synonyms.

Prerequisite Behaviors:

1. Subjective probability notions of sure, possible, and impossible events. (See objectives of Lesson 1.)
2. Listing of possible outcomes of a one dimensional finite sample space and specifying the number of outcomes in the sample space.
3. Specifying the number of outcomes for an event (I-D).

Introduction*

Up to this point pupils have gathered data from a variety of activities and have learned to summarize it in tables and to represent it visually through the construction of graphs. The probability or chance of various results has been discussed informally on the basis of intuition and hunches. The graphs and charts provide a systematic way of organizing data so that they can be studied and analyzed to discover relationships and patterns. These relationships and patterns enable us to formulate hypotheses and to draw various conclusions.

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*This lesson is taken from SMSG's Probability for Intermediate Grades pp. 83-88. (Teacher's Commentary).

This lesson begins with a review of the informal ideas pupils have gained from previous lessons. Pupils are then guided to compare the results of different activities; to identify those activities which produce like results and those which produce unlike results; and to discover the causes which may underlie their likenesses and differences. As a natural outgrowth of these comparisons, pupils are introduced to the use of rational numbers as measures of chance and then to the use of number sentences to express probability.

The Suggested Procedure outlines a logical sequence for developing the concepts in this lesson. You may choose to condense or extend the lesson to provide the pacing best suited to your pupils.

Vocabulary:

probability, number sentence, express, mathematician.

Materials to be used:

1. Charts and graphs from Activities 1-10.
2. Spinner (1/2 red, 1/2 blue).
3. Spinner (1/4 red, 3/4 blue).
4. Spinner (1/3 red, 1/3 blue, 1/3 yellow).
5. Colored marbles or beads (1 red, 1 white, 1 blue, 1 yellow).

Suggested Procedure:

Begin by having pupils recall the ideas developed during discussion of experimental activities. (Have charts and graphs available.) Ideas to be recalled would include:

1. Many events are uncertain.
2. Two events may or may not be equally likely.
3. We can never be certain of the exact outcome of chance events.
4. There may be a pattern in large numbers of chance events. This pattern can help us estimate what is likely to happen if the events are repeated.
5. We can use experimental activities to check our estimates about chance events.

> Comment:
>
> Since these ideas had been discussed in the last part of Lesson 3 which immediately preceded Lesson 4, this was not done as part of Lesson 4.

Show the spinner with dial 1/2 red and 1/2 blue. Ask questions such as:

What part of the dial is red? (1/2) Blue? (1/2) If 1/2 of the dial is red and 1/2 is blue, is there one chance in two of getting red? (Yes) Of getting blue? (Yes).

> How could we change the dial so there would be a better chance of getting red? (Make more of the dial red.)
> What will happen to the chance of getting blue if we increase the chance of getting red? (Chance of getting blue will be less.)

Show the spinner with dial 1/4 red and 3/4 blue. (Comment: Girl said, P (R) = 1/2 chances, since there are 2 outcomes. Class did not agree. The majority wanted to assign 1/4 and 3/4.) Ask questions as before to show that there is 1 chance in 4 of getting red and 3 chances in 4 of getting blue. Also:

> Which of these spinners would you choose if you wanted to get blue on one spin? (The one with the largest blue area.)
> Which of these spinners would you choose if you hoped to get about 50 red in 100 spins? (The first one--the one with equal amounts of red and blue.)

Show the spinner with dial 1/3 red, 1/3 blue, and 1/3 yellow, and discuss similarities and differences among the three spinners. Bring out that there is 1 chance in 3 of getting red, 1 chance in 3 of getting blue, and 1 chance in 3 of getting yellow.

Summarize by helping children see, for example, that if 1/2 of the dial is red means 1 chance in 2, then 1 chance in 2 means 1/2 of the dial is red. Carry them through the same idea with the spinner with dial 1/4 red and 3/4 blue and the spinner with dial 1/3 red, 1/3 blue, and 1/3 yellow.

Lead them to generalize that 1 chance in 2 of getting red means that the chance of red is 1/2 and that, in this case, 1/2 of the results are likely to be red. Allow sufficient discussion to develop understanding of this concept. Then ask such questions as:

> If a spinner dial is 2/3 blue and 1/3 red, what is the chance of getting red? (1/3, or 1 chance in 3.)
>
> What is the chance of getting blue? (2/3, or 2 chances in 3.)
>
> If the chance of getting red on a spinner is 1/5, what part of the dial is red? (1/5)
>
> What part is not red? (4/5)
>
> Then what is the chance of getting a color other than red on such a spinner? (4/5, or 4 chances in 5.)

Continue until children understand the relationship between the chance of getting a given color and the fraction of the dial that is covered by that color. Then ask:

> If 1/5 of a dial is red, what is the chance of getting red? (1 out of 5, or 1/5.)
>
> If 4/5 of a dial is red, what is the chance of getting red? (4 out of 5, or 4/5.)
>
> If 5/5 of a dial is red, what is the chance of getting red? (5 out of 5, or 5/5, or 1.
>
> Children should generalize that when a result is certain to happen, the chance of that result is

1 out of 1, or 1/1, or 1. Thus, if the chance of red equals 1, we know that red must occur and that red is the only result possible in this case.)

Let us look at these ideas another way.

If 4/5 of a dial is red, then the chance of red is 4 out of 5, or 4/5.

If 1/5 of a dial is red, then the chance of red is 1 out of 5, or 1/5.

Now, think about this. If no part of a dial is red, then what is the chance of red? (There is no chance of red because there is no red on the dial.)

What number could we use to mean there is no chance of getting red? (Zero. Children should generalize that when a result cannot occur, the chance of that result is zero, or 0. Thus, if the chance of red equals 0, we know that red cannot occur and that the result, red, is impossible in this case.)

At this point, help children summarize these ideas. The summary should include the following points in the pupils' own words.

1. We can use fractions to compare the chances of different results. (Do this by asking, how do you describe the number that we assign to the uncertain event?)
2. If some result is certain to happen, we say the chance of that result is equal to one. (Do this by asking, probability of the certain event is equal to what?)

3. If some result cannot happen, we say the chance of that result is equal to zero. (Do this by asking, probability of the impossible event is equal to what?)

Have pupils open their texts to page 38 and complete it as a class activity. Discuss it as necessary and then do pages 39, 40, and 41 together and have the children discuss their answers. [Comment: Done in class, collected and graded.]*

Do not go on to Pupil page 42 yet but introduce the last part of this lesson in a way similar to this: [Comment: This was done the following day.]

> From our activities and discussions, we have discovered several ideas about chance. We can use numbers to describe the chance that some event may or may not occur. For instance, if an event is certain to occur, the chance that it will occur is equal to one. [Comment: Children were asked for instances of certainty, uncertainty, and impossibility. These were discussed in detail and were accepted or not accepted by students. Probabilities were assigned by the children to the instances of certainty and impossibility.] If an event cannot occur, the chance that it will occur is equal to 0. And, if an event is uncertain, the chance that it will occur is equal to some fraction between 0 and 1. Mathematicians use the word probability in much the same way that we have used the word chance. (Write

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*Exercise based on pp. 38-45 of Probability for Intermediate Grades.

probability on the chalkboard.) We know that we can use a number to describe the chance that an event will occur. This number is called the probability of the event. Where we have said, "The chance of red is equal to 1/2", a mathematician would say, and you may say, "The probability of red is equal to 1/2."

In mathematics we use a few letters, numerals, and signs to stand for a big idea that we would otherwise have to use many words to explain. (Show spinner with 1/2 red, 1/2 blue dial.) If we asked a mathematician to describe the chance of red on this spinner, he would say, "The probability of red is 1/2", and he would write P(R) = 1/2. (Write "P(R) = 1/2" on the chalkboard. Ask what the "R" represents.) From now on we will use just the first letter instead of writing out the name of the color.

Here are two marbles, one red and one blue. If I place them in my pocket and then pick one without looking, how could we describe the chance that it will be blue? That is will be red? Write on the board, P(B) = 1/2, and, also, P(R) = 1/2. [Comment: Students came and wrote each of these on board and then read them from the board.]

Suppose I add a yellow marble so that there are a red, a yellow, and a blue. What is the probability of picking a blue? (John, will you write the number sentence on the board? [P(B) = 1/3] A red? [P(R) = 1/3] A yellow? [P(Y) = 1/3]

I will add the white marble. Now there are a red, a white, a blue, and a yellow. How could we describe the chance of picking a yellow? May, please write it on the board. [P(Y) = 1/4] A blue? [P(B) = 1/4]

What is the probability of picking either a red or a white? [P(R or W) = 2/4 = 1/2] (Here we want either one of two of the four possible events.)

What is the probability of picking a green? [P(G) = 0. Whenever an event is impossible, its probability is 0.]

What is the probability of picking either a red, a white, a blue, or a yellow? [P(R or W or B or Y) = 1. Whenever an event is certain to occur, its probability is 1.]

Now we have a way of using numbers to express probability. Let's complete Pupil page 42 through 44. Many of you will be able to answer the Brain Teasers on page 45.

After the boys and girls have completed these pages, discuss their answers. On Pupil page 42, the answer expected to the last two items in the first column is 2 red marbles and 1 blue marble, but it could be 4 red marbles and 2 blue marbles, etc.

Put 3 blue and 3 red marbles in a box and ask "How many possible outcomes?" "How many outcomes give you blue?" "What's the probability of blue, P(R), P(R or B)?" Then add 3 red. "How many possible outcomes?" "What's the probability of red, P(B), P(R or B)?" "What's the probability of green?" Then add 1 white, "Possible outcomes?" (10) "What's the P(W), P(B)?"

[Comment: Again the teacher called on students and the children wrote each of these on the board and read their answers, translating the symbols.]

INTRODUCTORY DISCUSSION OF LESSON 4

The major goal of Lesson 4 is to use numbers ($o \leq a/b \leq 1$) to express the probability of an event in one-dimension. An exercise accompanies the lesson in order to give students practice in assigning probabilities.

Lesson 4 was done in two parts. The first part was done on Wednesday and lasted 35 minutes. It covered pages 83-86 of the lesson plus pages 38-41 of the exercises for Lesson 4.

The rest of the lesson was completed on Thursday. The exercise for Lesson 4 was to be completed as homework for Friday.

Since the dice game from Lesson 1 had not been used, it was decided to use it as a way to introduce the second part of Lesson 4.

The teacher asked the class a number of questions. Only the wrong responses or particularly interesting responses are noted in parentheses after a question.

Discussion of Lesson 4--Wednesday, March 12

The first part of Wednesday was spent finishing the presentations of graphs by students (twelve minutes). The rest of the period was spent on Lesson 4, which is concerned with an introduction to probability in one dimension. The probability of an event in one dimension is approached through concrete models. Assigning a fraction to the probability of an event including 0 to the impossible event, and 1 to certain event is the major objective of the lesson. An exercise for Lesson 4 is employed to give the student practice in using these concepts.

Discussion of Lesson 4--Thursday, March 13

The dice game from Lesson 1 was used to introduce the second part of Lesson 4. It was played 22 times the first time, with each person taking a turn throwing the dice into a box that the teacher held as she walked around the room. Two students acted as recorders at the board. Results were 15 - teacher; 7 - students.

The students wanted to play again. This time she said she would give them any number from her list (5, 6, 7, 8, 9) and they could give her any number from theirs (2, 3, 4, 10, 11, 12). They picked the "7." There was a debate over whether to give her a "2" or a "3." The "2"'s won out. However, when "2" came up more often than "3" for the teacher those in favor of giving her the "3" said: "See, we told you." Results were 12 - teacher; 8 - students.

The students were very enthusiastic and cheered quite loudly whenever they had a winner.

After the second game, a student asked, "Are they loaded dice?" Another student said, "They wouldn't use loaded dice." A boy said, "There are more combinations that make you a winner." Another boy said, "There are more ways to get 4, 5, or 6 than 2, 3, or 12."

At this point the teacher pointed out that the understanding of this game is a goal of the following lessons. The game took 10 minutes.

The lesson then proceeded according to the lesson plan.

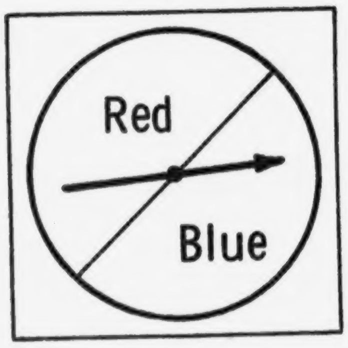
When the handout for the lesson was passed out, problems from yesterday's work (pp. 38 - 41) were discussed. The question of the meaning of "number sentence" was raised by a student. The teacher said that $P(R) = \frac{1}{2}$ is an example of a number sentence.

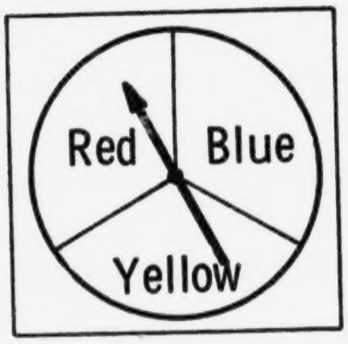
Comment:

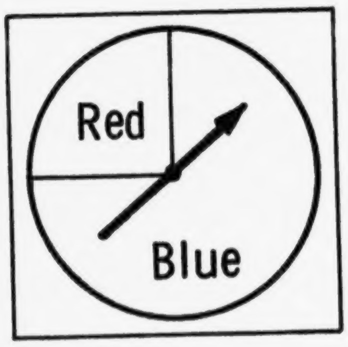
This should have been mentioned before.

Due to the math period ending 5 minutes before the scheduled time because of a school activity, there was only 5 minutes for the students to start the handout. They were instructed by the teacher to finish the exercise for Friday.

Exercise 4.1*







Look at these spinners.

You know that you can use fractions to compare the chances of different results.

Complete this table.

$\frac{1}{2}$ of dial red	means	1 chance in 2	means	Chance of red = $\frac{1}{2}$
$\frac{1}{2}$ of dial blue	means	___ chance in 2	means	Chance of blue = ___
$\frac{1}{3}$ of dial red	means	1 chance in ___	means	Chance of red = ___
$\frac{1}{3}$ of dial blue	means	___ chance in 3	means	Chance of blue = ___
$\frac{1}{3}$ of dial yellow	means	___ chance in ___	means	Chance of yellow = $\frac{1}{3}$
$\frac{1}{4}$ of dial red	means	___ chance in ___	means	Chance of red = ___
$\frac{3}{4}$ of dial blue	means	___ chances in 4	means	Chance of blue = ___
All of dial red	means	red is certain	means	Chance of red = ___
___ of dial red	means	red is impossible	means	Chance of red = 0

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*Taken from pp. 38-45 of Probability for Intermediate Grades

Exercises - Lesson 6.

1. James spins the pointer of a spinner 100 times and gets 35 reds. Which of the following statements is most likely to be true?

   (a) The dial of the spinner is all red.

   (b) The dial of the spinner is one-half blue.

   (c) The dial of the spinner is one-eighth red.

   (d) The dial of the spinner is one-third red.

2. Mary spins the pointer of a spinner 100 times and gets 25 red, 25 blue, and 50 yellow. Which of the following statements cannot be true?

   (a) The dial of the spinner is one-fourth yellow.

   (b) The dial of the spinner is one-third green.

   (c) The dial of the spinner is one-fourth blue.

   (d) The dial of the spinner is all red.

3. A spinner has a dial that is one-third red, one-half white, and one-sixth blue. Which of the following cannot result from exactly 100 spins?

   (a) 30 reds, 50 whites and 20 blues.

   (b) 40 reds, 40 whites and 20 blues.

   (c) 50 reds, 5 whites and 10 blues.

   (d) 60 reds, 40 whites and 0 blues.

4. You wish to get exactly 5 reds and 5 blues in 15 spins. Which of the following dials could not give this result?
   (a) One-half red and one-half blue.
   (b) One-third red, one-third blue and one-third yellow.
   (c) One-fourth red, one-fourth blue and one-half yellow.
   (d) One-fifth red, two-fifths blue and two-fifths yellow.

5. In which of the following statements is the chance of red equal to $\frac{1}{4}$?
   (a) One chance in two of red.
   (b) Two chances in four of red.
   (c) One chance in five of red.
   (d) Two chances in eight of red.

6. Which of the following spinners is likely to give about the same number of reds and yellows?
   (a) One-half red, one-fourth yellow, one-fourth blue.
   (b) One-third red, two-thirds yellow.
   (c) One-third red, one-third yellow, one-third blue.
   (d) Four-fifths yellow, one-fifth red.

7. If the dial of a spinner is all red, we say the chance of red is equal to:
   (a) any other chance.
   (b) one chance in two.
   (c) one-half.
   (d) one.

8. If the dial of a spinner is all blue, we say the chance of red is equal to:
   - (a) one.
   - (b) zero.
   - (c) one chance in one.
   - (d) one-half.

9. The dial of a spinner is one-third red, one-third yellow, and one-third blue. Which of the following statements are true?
   - (a) Red, yellow, and blue are equally likely to occur.
   - (b) The chance of getting red is equal to $\frac{1}{3}$.
   - (c) One spin must result in either red or yellow or blue.
   - (d) The chance of getting green is equal to zero.

10. If the chance of red on a spinner is equal to zero, which of the following statements could be true?
    - (a) The dial is all red.
    - (b) The dial is all blue.
    - (c) The dial has at least two colors.
    - (d) The dial has at least three colors.

11. Complete this table.

All of dial red	means	red is certain	means	Chance of red = 1	means	P(R) = 1
None of dial red	means	red is impossible	means	Chance of red = 0	means	P(R) = ___
$\frac{1}{2}$ of dial red	means	1 chance in 2 of red	means	Chance of red = ___	means	P(R) = ___
$\frac{1}{2}$ of dial blue		___ chance in 2 of blue		Chance of blue = ___		P(B) = ___
$\frac{1}{4}$ of dial red	means	1 chance in ___ of red	means	Chance of red = $\frac{1}{4}$	means	P(R) = $\frac{1}{4}$
$\frac{3}{4}$ of dial blue		___ chances in ___ of blue		Chance of blue = ___		P(B) = ___
$\frac{1}{3}$ of dial red	means	___ chance in 3 of red	means	Chance of red = ___	means	P(R) = ___
$\frac{1}{3}$ of dial blue		1 chance in ___ of blue		Chance of blue = ___		________
$\frac{1}{3}$ of dial yellow		___ chance in ___ of yellow		Chance of yellow = ___		________
___ red cubes and	means	2 chances in ___ of red	means	Chance of red = ___	means	________
___ blue cube		___ chance in 3 of blue		Chance of blue = $\frac{1}{3}$		P(B) = ___

12. A spinner has a dial which is evenly divided into red, white, and blue spaces. Write a number sentence that describes the chance of getting blue. ____________________

13. Write a number sentence that answers the question, "What is the probability of yellow on the spinner in Problem 12 ?" ____________

14. In Problem 12, P(R) = __________.

15. Write this number sentence (about Problem 12) in words: $P(W) = \frac{1}{3}$.

____

16. A bag contains several marbles. Some are red, some white, and the rest blue. If you pick one marble without looking, the probability of red is $\frac{1}{3}$ and the probability of white is $\frac{1}{3}$. What is the probability of blue?

____

17. A bag contains one red marble, two white marbles, and three blue marbles. If you pick one marble without looking, what is the probability that the marble will be red? __________

18. In Problem 17, what is the probability that the marble will be white? ___________

19. In Problem 17, what is the probability that the marble will be blue? ___________

20. In Problem 17, how many white marbles must be added to the bag to make the probability of white equal to $\frac{1}{2}$? ___________

21. Write the following number sentence in symbols: "The probability of yellow is equal to three-fourths." ________________

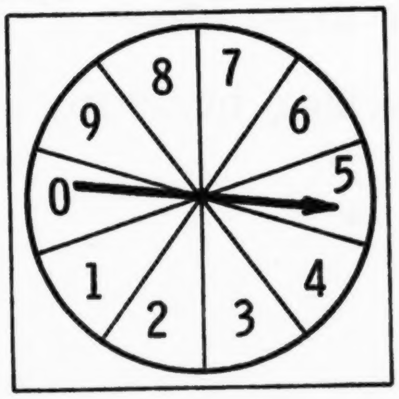
22. A wooden cube has a dot on two of its faces. If it is tossed on the floor, what is the probability that a face with a dot on it will be on the bottom when it stops rolling? ____________

23. In Problem 22, what is the probability that a face without a dot will be on the bottom? ____________

24. The dial of a spinner is divided into three colors: red, white, and blue. If $P(R) = \frac{1}{2}$ and $P(W) = \frac{1}{4}$, what is the probability of blue? ____________

25. In Problem 24, is the probability of red greater than, less than, or equal to the probability of blue? ____________

The dial of this spinner is divided into 10 equal regions.

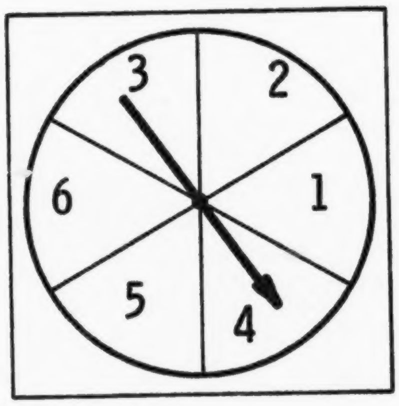


26. P(3) = ________.

27. P(10) = ________.

28. Is P(4) = P(8) ? ________

The dial of this spinner is divided into 6 equal regions.



29. P(2) = ________.

30. P(5) = ________.

Brain Teasers

1. John has ten pairs of socks in a drawer. Nine pairs are red and one is blue. If he picks the socks one at a time without looking, how many socks must he pick to be sure he has two socks of the same color? _______

2. A bag contains several marbles. Some are red, some white, and the rest blue. If the probability of picking red is $\frac{1}{6}$ and the probability of picking white is $\frac{1}{3}$, what is the probability of picking blue? _______

3. In Brain Teaser 2, what is the smallest number of marbles that could be in the bag? _______

4. In Brain Teaser 2, could the bag contain 48 marbles? _______

5. In Brain Teaser 2, if the bag contains 4 red marbles and 8 white marbles, how many blue marbles does it contain? _______

COMMENT ON EXERCISE FOR LESSON 4

The exercise was passed out in class Wednesday and was collected at the end of the peiod. It was handed out Thursday as homework for Friday. The exercise was discussed on Friday for the first ten minutes of the period.

In discussion of the homework for Friday it was clear that certain questions were missed frequently. For problem 13, page 43, "What is the probability of yellow on the spinner in problem 12?" Many wrote P(y) = 1/3 rather than 0. Problem 27 was another item commonly missed (P(10) = 1/10 rather than 0). Both of these mistakes were probably the result of not carefully considering the model involved in the question and the poor format of the question.

Lesson 5

PROBABILITY OF AN EVENT

Two-Dimensional Finite Sample Space

Objectives: The child should be able to:

1. Identify a 2-tuple as a sample outcome.
2. List possible outcomes from a two-dimensional, ordered, finite sample space.
3. Specify the number of possible outcomes for a two-dimensional, ordered, finite sample space.
4. Specify the number of outcomes of an ordered event.
5. Specify the probability of an outcome from a two-dimensional, ordered, finite sample space.
6. Specify the probability of an ordered event in a two-dimensional, ordered, finite sample space.

Prerequisite Behaviors:

1. Specifying the probability of an event (1-D)
2. Using the appropriate probability notation

Materials to be used:

1. Spinners
2. Marbles
3. Dice
4. Handout

New Vocabulary:

None.

Method of Presentation:

Review p (E) = number of favorable outcomes/ number of possible outcomes by taking a box with seven red and three white marbles, (one draw), and asking (1) P(R) = ? (7/10)

Review Notation:

P(R) = 7/10

1. P stands for probability.
2. ( ) means "of."
3. R means Red.
4. = means is
5. 7/10 means seven tenths or seven out of ten.

Then Ask:

(2) P(R or W) = ? (3) P(Y) = ? (0)

Review the Generalization of these Results by Asking Class:

How do I find the probability of an event such as getting a red marble in the previous problem? (number of favorable outcomes/number of possible outcomes). What is the probability of the sure or certain event? (1)

What is the probability of the impossible event? (0) What can you say about the probability of the uncertain event? (fraction between 0 and 1)

Show model with two spinners and ask class to tell you what the probability of white on the first spinner and yellow on the second spinner is if you spin each of them once. (The probably will say 2/4--ask why?) If no child proposes 1/4 as an alternative (Comment: No one did), then say: Let's look at it a different way. Suppose you're a winner if you get Red on the first spinner and Yellow on the second spinner. How many ways can you win? (Comment: Teacher said how many ways first, then, how many winners and losers.) (1) Let's list all the ways we can lose. (3) How many ways is that altogether? (4) What is the probability of winning (i.e., WY)? (1/4) Is it 2/4 or 1/4? How could we find out which one is right.

The student may suggest carrying out an experiment say 40 times. If not, suggest it. Have a student record results to show them that the data does not agree with P(RY) = 2/4. Ask "what would you expect to get on the average for P(RY) = 2/4, P(RY) = 1/4 in 40 trials? Which does the data support?" Bring out that there is a need to look closely at such problems in order to count the number of favorable outcomes and the number of possible outcomes.

Stress that WY (white, yellow) is only one outcome.

Give students handout. Go over the results. (Comment: The first problem and part of the second were done together.)

Give students quiz. [Comment: No time for quiz.]

Lesson 5*

FRIDAY, MARCH 14

1. Go over pages 42-45 of homework from Lesson 4 (10 minutes).
2. Do Lesson 5 (12 minutes).
3. Pass out Handout on Lesson 5 (13 minutes).
4. Give Quiz on Lessons (1-5) (15 minutes).
5. When children finish the quiz, they are to correct their mistakes on bar graphing.

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*The teacher used a handwritten outline on a 3 x 5 paper like the one above to teach from. The same comment applies to the outlines of Lessons 6-9. The author has taken the liberty of adding a few points on the outlines of Lessons 5-9 where a verbal agreement to do certain activities had been made.

DISCUSSION OF LESSON 5

Friday, 3/14

Since the first 20 minutes of Friday's session was spent on the discussion of the exerices from Lesson 4, the teacher omitted the planned review.

The rest of the lesson proceeded as planned.

With regard to the model with two spinners [(1/2 R, 1/2 W) and (1/2 B, 1/2 Y)], when the question, "what is the probability of white on the first spinner and yellow on the second spinner?" no student initially suggested 1/2 as an alternative. The initial answer given by the students was 2/4. After listing the four otucomes a student said the probability was 1/4. A student's reply to the question, "Is it 2/4 or 1/4? How could we find out which one is right?", was to say that there are four outcomes and there's one way to win so it has to be 1/4. No student suggested carrying out an experiment to find out whether the P(RY) was 1/4 or 2/4.

The teacher suggested the experiment. It was decided the day before the lesson to perform the experiment 24 times. The question was asked by the teacher as to what would you expect to get (in 24 times), on the average, if P(RY) = 2/4? They said 12 without any problem. However, for the same question asked with regard to P(RY) = 1/4, only a few hands went up. A boy answered, "6."

The teacher took the spinners around the room while a student tallied the number of WY, WB, RB, and RY on the blackboard.

The results were:

WY	5
WB	6
RB	8
RY	5
	24

The children's favorite in doing the experiment was RB. They cheered each time it came up. The teacher asked which probability does the data support. They agreed 1/4. She also tried to get them to verbalize that each of the outcomes were equally likely. She finally made the statement herself and that the data supported this since all the results were around 6. The teacher asked what P(RY) = ? They said 5/24. One boy then wanted to round this off and said, "5/24 is close to 1/5."

Comment:

The *a posteriori* probability estimate based on the data seemed to be a more natural response than looking at the outcomes and assigning the *a priori* probabilities. Also, it was not clear whether the students believed that each of the four outcomes are really equally likely.

The exercise for Lesson 5 was distributed and the first problem and part of the second were worked in class. (See exercise for comments concerning this.) There was no time for Quiz 1 that was planned to measure certain objectives of Lessons 1-5. Besides, the students were obviously confused by counting a two-dimensional outcome as one outcome. It was decided that the students would need further practice before being tested on the objectives of Lesson 5.

EXERCISE 5.1

Lesson 5

John and Paul each have one white and one green marble. John picks one of his marbles without looking and then Paul picks one of his. The four possible outcomes are listed in the table below. Complete the table on the right to show the outcomes in a shorter way.

	John's Marble	Paul's Marble
1.	White	White
2.	White	Green
3.	Green	White
4.	Green	Green

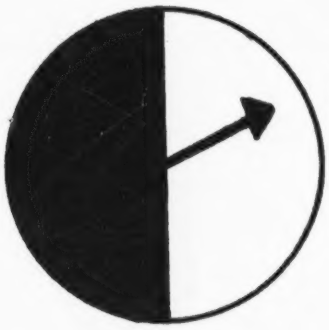
	John's Marble	Paul's Marble
1.	W	W
2.	W	_
3.	G	_
4.	_	_

1. What is the probability that John picks a white marble? _______
2. What is the probability that Paul picks a white marble? _______
3. What is the probability that both boys pick white marbles? _______
4. What is the probability that both boys pick green marbles? _______
5. What is the probability that the boys pick a marble of the same color? _______

This problem taken from SMSG's Probability for Intermediate Grades.

Lesson 5

You spin the spinner at the right two times. List the possible outcomes.



	1st spin	2nd spin
1.	W	W
2.		
3.		
4.		
5.		

1. 1. How many possible outcomes are there? _____ How many ways can you get white on the first spin and white on the second spin? _____

2. What is the probability of getting white on the first spin and white on the second spin? ________

3. What is the probability of getting black on the first spin and any other color on the second spin? ________

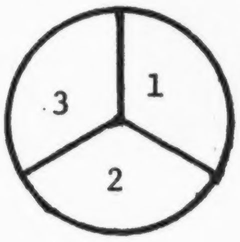
4. What is the probability of getting black on the second spin and any other color on the first spin? ________

5. What is the probability of getting BW or WB? ________

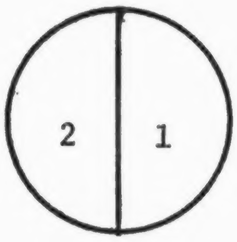
Lesson 5

You spin each of the spinners below one time.

1st Spinner



2nd Spinner



List the possible outcomes

	1st Spinner	2nd Spinner
1.	3	2
2.	3	1
3.		
4.		
5.		
6.		

1. How many possible outcomes are there? ________
2. How many ways can you get "3" on the 1st spinner and "2" on the second spinner? ________
3. What is the probability of getting a "3" on the 1st spinner and a "2" on the second spinner? ________
4. What is the probability of getting a "3" on the first spinner and any number on the 2nd spinner? ________

DISCUSSION OF EXERCISE (LESSON 5)

Problems (1-5) on page 1 and problems (1-3) page 2 were covered in class. The children had a difficult time accepting that the first problem on page 1 had 4 outcomes. They wanted to say that there were 8 outcomes.

The same response occurred with the stem on page 2. However, the teacher said "Look WB is one outcome." The students then agreed that there were 4 outcomes. The students completed the exercise in class. The exerices was collected and the problems on page 2 and page 3 were corrected. Of the 22 people completing the exercise only 8/22 were classified as masters (0 or 1 mistake). The mean on the 10 items was 7.68 and the variance 2.94. The item difficulties for these items were:

Page 2	1.	a.	22/22
		b.	21/22
	2.		22/22
	3.		16/22
	4.		15/22
	5.		15/22
Page 3	1.		19/22
	2.		19/22
	3.		12/22
	4.		8/22

Students missing 4 or more were subjects 9, 13, 14, 15, 20 and 22. The students were instructed to correct their mistakes and show them to the observer.

On Problem 4, page 2, "What is the probability of getting black on the second spin and any other color on the first spin?" The accepted answer was 2/4. However, as was discovered at a later time, the answer should be 1/4. The author was thinking of the question "What is the probability of getting black on the second spin and any color on the first spin?"

In class, 2/4 was the only accepted answer. (5/7 who missed this put 1/4 for their answer). Problem 3, page 2 has the same problem in wording. On page 3, Problems 3 and 4 caused a great deal of difficulty. In regard to Problem 3 the wrong responses were: 1/5, 1, 1/4, 1/3 and 1/2, 1/5, 1/2, 2/5, 2, 1/6 and 1/6, 1/5. Problem 4 had the following wrong responses: 1/6, 1/6, 1/4, 1/2, 2, 1/2, 1/3, 1/2, 2/5, 3/5, 1/6, 2, 1/2, 1/6 and 3/6, 1/5. Problem 4 asks "What is the probability of getting a "3" on the first spinner and any number on the second spinner?" This question is probably too difficult for an introductory lesson in two-dimensional problems.

There is no doubt that quite a few students were confused by two-dimensional probability problems. They tend to view a two-dimensional outcome as two outcomes.

The entire handout probably should have been done as a class activity that was completed and corrected together. Many students were not ready to answer these types of questions by themselves. Problems 1-5 on page 1 should probably have been omitted or done last since they are based on a word problem without a picture.

Due to students' problems with two-dimensional problems, it was decided to postpone the first quiz until after Lesson 6 had been introduced. This would enable the students to have more practice with two-dimensional problems.

OVERVIEW OF LESSON 6

Parts I and II

Lesson 6, Parts I and II, were scheduled to take three days. However, the first session of Lesson 6, Part I, on Monday (3/17) included 20 minutes for Quiz I on the objectives of Lessons 1-5. Most of Exercise 1 (Lesson 6) was completed in class.

From an analysis of the quiz it became apparent that more emphasis and practice were needed on certain objectives of Lessons 1-5. Part of Tuesday's session (3/18) was spent doing this. The rest of Tuesday's session was concerned with working on Exercise II (Lesson 6).

The first part of Wednesday's session was spent going over Exercise II. The rest of the period centered on Lesson 6, Part II.

The first part of Thursday's session was spent going over Exercise III. A quiz (Quiz IIA and Quiz IIB) was then administered that tested the objectives of Lessons 1-5 and Lesson 6. Based on an analysis of the mistakes on Quiz IIB, Lesson 6, part of Friday's and Monday's sessions were used to give the students more practice. A second quiz on Lesson 6 was administered on Monday. On Tuesday (3/25), since 9/25 children were still nonmasters of certain objectives of Lesson 6, the class was split into masters and nonmasters. The masters worked on Exercise II of Lesson 7. The nonmasters were given more treatment concerning the objectives of Lesson 6 in a small group situation. Subsequently, 3/9 were classified as masters. At this point

Lesson 6

COUNTING PROCEDURES (PARTS I AND II)

Objectives: The child should be able to:

1. List the possible outcomes of an experiment by employing a tree.
2. List the possible outcomes of an event by employing a tree.
3. Count the possible outcomes of an experiment.
4. Count the possible outcomes of an event.
5. Specify the probability of an event after employing a counting paradigm (2 or 3 D. sample space).

Prerequisites:

1. Specify the probability of an event.
2. Specify the cardinality of a given set of outcomes from an experiment.

Materials used:

1. Spinner (1/2, 1/2)
2. Marbles (3 r, 3 b), 3 boxes
3. Dice

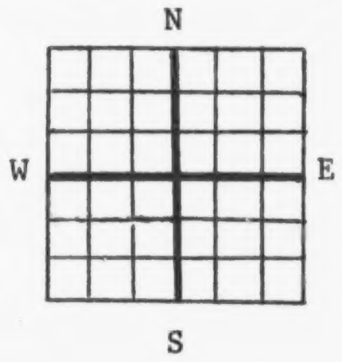
New Vocabulary:

tree, table, row, column.

19/25 were considered masters of the objectives of Lesson 6. It was decided that no more class time would be spent on Lesson 6. However, the six nonmasters were given further help Wednesday after class concerning Lessons 6 and 7.

Method of Presentation:

Use the spinner (N, S, E, W), a graph on an overlay, and a marker as in the picture at the right. Explain to the children that "we are going to play a game on the overhead using the Spinner (N, S, E, W). The outcomes on the spinner stand for North (N), South (S), East (E), and West (W).

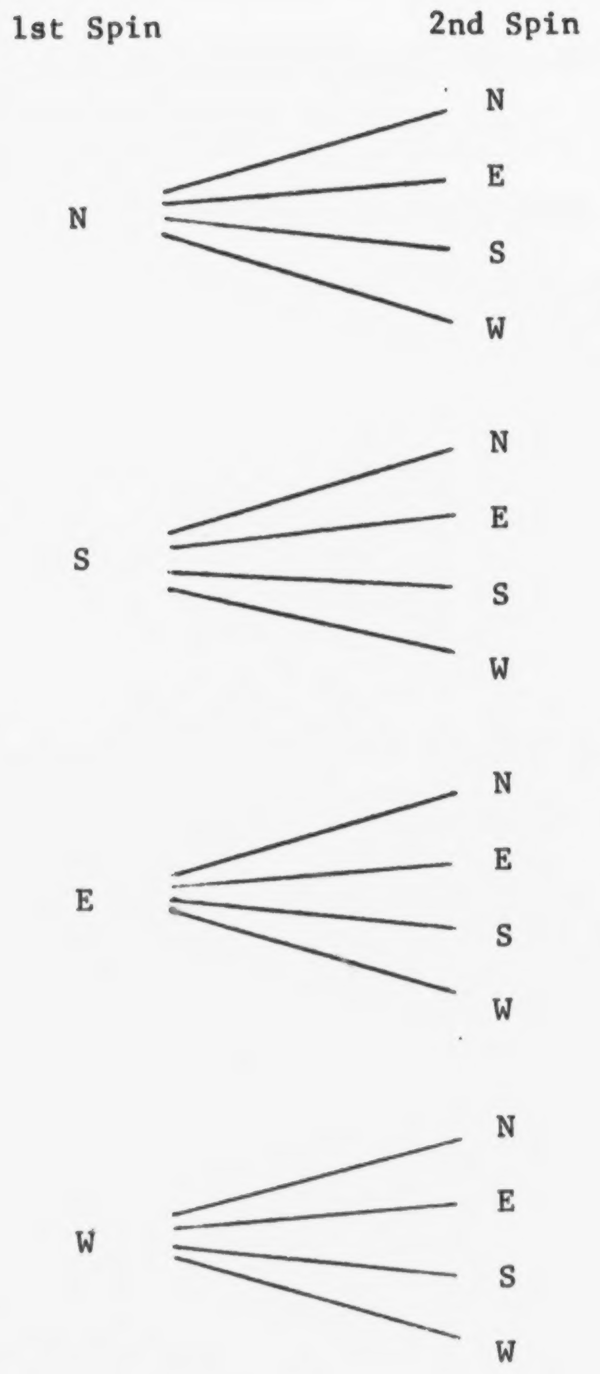


We move the marker along a line one space in the direction of the outcome gotten on the spinner. We begin at the middle of the graph." Via class discussion consider the following. "Suppose you play a game where you spin the spinner only once. You win if you move one square North. What is the P(N)? (1/4) (There is one way to get N and there are four outcomes). Suppose now we play a game where we spin twice and we win if we get North on the first spin and North on the second spin. Let's abbreviate that outcome NN. What is our chances of winning (i.e. What is P(NN)?) How many ways are there to win ? (1) How many ways can the marker be moved two squares? Let's list some (on the board). Do you think you have them all (There are 16)?"

"There is a very good way of counting outcomes by using what is called a 'tree.'"

"Consider the following.

What can happen on the first spin?"



"'If we get N on the first spin, what could we get on the second spin?"
(N E S W)

"How about if we get S on the first spin?"
etc.

"How many ways are there? Let's list them."

1. NN
2. NE
3. NS
4. NW
5. SN
6. SE
7. SS
8. SW
9. EN
10. EE
11. ES
12. EW
13. WN
14. WE
15. WS
16. WW

DND*

"What is P(NN)? (1/16) Suppose we win if we get NN or SS. What is P(NN or SS)? (2/16) Suppose we win if we get to the edge of the first square in two moves? What is the P(Winning)? (4/16) The "tree" we used is a very good way to list all the possible outcomes for a difficult situation." DND

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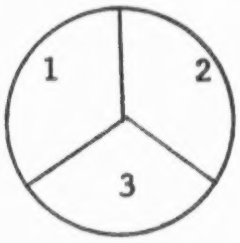
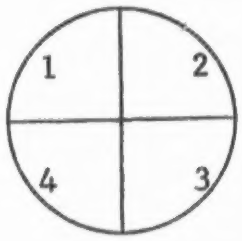
*Did Not Do

"Let's practice using it. Suppose I spin the two spinners below once." [Comment: The teacher drew these spinners on the board.]

1st Spinner

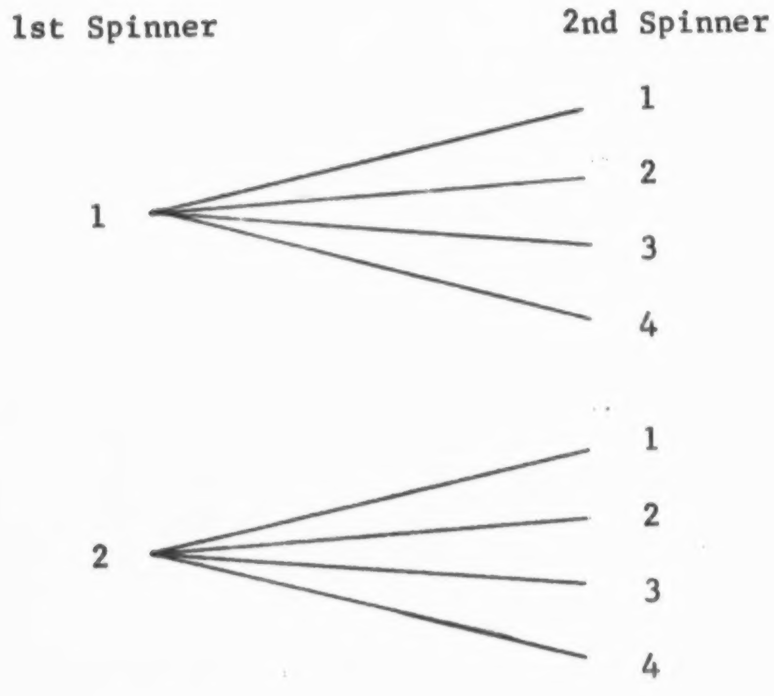


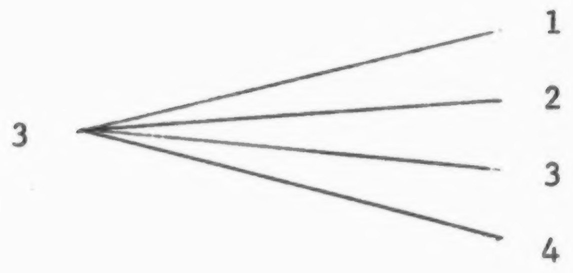
2nd Spinner



"How many outcomes are there? Let's use a tree to find out. What do we do? What can we get on the first spinner?"

"On a piece of paper let's see if you can use a tree to list the possible outcomes." (Have a student who gets it right put it on the board.)





"Let's count the outcomes."

1. 1,1
2. 1,2
3. 1,3
4. 1,4
5. 2,1
6. 2,2
7. 2,3
8. 2,4
9. 3,1
10. 3,2
11. 3,3
12. 3,4

Comment:

The teacher listed the outcomes after doing the probability problems below. She did not place counting numbers before the outcomes.

"How many possible outcomes are there? (12)

What is the P(2,2)? (1/12)

Comment:

The teacher also did the problem: P(4,3) = ? (0)

What is the P(2, 2 or a 1,1) (2/12)

Suppose we add the two numbers together, what is P(sum = 2)? (1/12).

What is P(sum = 4)? (3/12). P(sum = 3 or 4)? (5/12). P(sum is less than 10)? (1)."

Comment:

The teacher also did the problem: P(sum is greater than 6.)

Pass Out Handout 1--Lesson 6

"Now I am going to give you some sheets which will give you practice in using a tree to find the number of outcomes and to do probability problems.

"Let's do #1 together. Fill in the blanks and then the table; you may use abbreviations to make your work easier. (Go over problem)"

Have them do the next couple of problems by themselves. Go over the problems. Point out to them the table is only a way to help them in counting and listing the outcomes. If they can count and list possible outcomes from the "trees," there is no need to make a table. Try Problem 4 without using a table.

"Now the rest of the problems are to be finished for tomorrow. You may use the rest of the period to answer the problems."

Lesson 6 - Part II

1. Go over homework.

2. Introduce dice by analyzing the dice game where you take the sum 5, 6, 7, 8, 9, and they take 2, 3, 4, 10, 11, 12. Point out to them that they have one more sum than you.

Do you remember when playing the game with the dice? I took 5, 6, 7, 8, 9 and you took 2, 3, 4, 10, 11, 12 and even though you had one more sum than I, I won. Here are the results: Me = 15 You = 7. Can you think of any reason why I won? If we did it again, each taking the same sums, do you suppose you could win? Why not? What are the chances of winning with each sum? Are they equally likely? How many outcomes are there? Total possible outcomes? (You should win since your chances are 24/36 and their's is 12/36.)

Pass out sheets that will help them list outcomes from dice and give them practice in doing problems. Have them do these at their seats. Discuss dice problems when they are completed. Look at 4 (h) and (i) in particular and ask them again why they think you won the game at the beginning of the period. (Because 24/36 > 12/36 and thus you have more chances of winning.)

Have them complete the rest of the problems for tomorrow.

Comment:

This was done the following day as the exercise was discussed.

Lesson 6 Part I

MONDAY, MARCH 17

1. The observer is to draw the grid on the board.
2. Review the goals of Lessons 4, 5 and 6 that are on the Goal chart.
3. Do Lesson 6.
4. Pass out Exercise 1. Do the problems together in class.
5. Give Quiz I.

DISCUSSION OF LESSON 6 - PART I

Monday, 3/17

This lesson is concerned with counting the number of possible outcomes by using a tree and the application of this technique to probability problems. Due to the previously mentioned difficulties with the overhead, the grid mentioned in the lesson was drawn on the board by the observer. While this was being done the teacher passed out the corrected handout for Lesson 5. The nonmasters were instructed to make their corrections and show them to the experimenter. The teacher then went over the goal chart showing the goals that had been accomplished and the ones with which Lesson 6 would be concerned. The lesson then proceeding as planned.

With regard to the spinner used to introduce the lesson, the students had no problem in assigning a probability to an event in which the spinner was spun once. When the spinner was spun twice the class was able to list only 15 of the 16 possible outcomes because of no systematic way of listing. (Given more time they might have found the 16th outcome.)

The first two pages in the exercise for Lesson 6, Part I, were done in class. The last page was assigned as homework and was discussed the following day. The only problem the children seemed to have was with the brain teaser.

The quiz was given at the end of the period. This was done by intent because of the students' problems with the exercise in Lesson 5 involving concepts of two-dimensional outcomes. The quiz took 13 minutes.

COMMENT ON LESSON 6

Part I

The teacher did not stress the process of taking the entries from a tree and making a list in a table form with the appropriate counting number beside each outcome.

e.g., 1. NN 2. NE 3. NS 4. NW etc.

In fact, with regard to the second problem introduced in the lesson, she asked them to find the probabilities from the tree. After doing this she made a list

(1, 1)	(2, 1)
(1, 2)	(2, 2)
(1, 3)	etc.
(1, 4)	

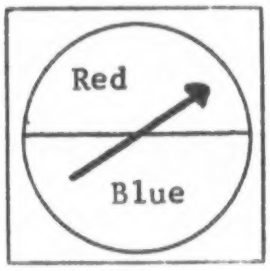
but she did not place a counting number beside each outcome. She told them that if they could do the problem without a table not to use it.

The author feels that it is important for the average student to go through the planned procedure first. Later, the listing and numbering of the outcomes from the tree can be dropped. But first the students must understand that an outcome such as (2, 1) is one outcome.

EXERCISE 6.1 (LESSON 6, PART I)

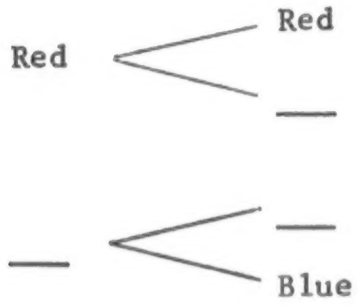
Exercise

1. Complete the tree diagram and the table to show the possible outcomes of two spins with this spinner.



Tree

First Spin Second Spin



Table

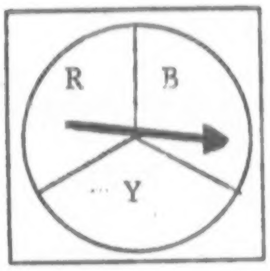
1.
2.
3.
4.

a. P(RR) = _________.

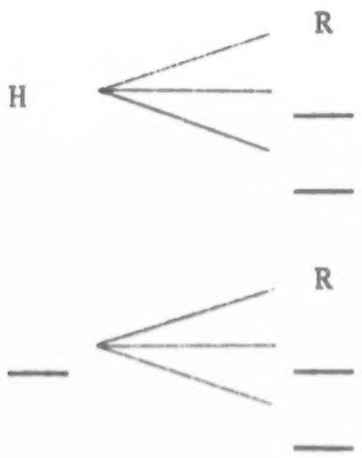
b. P(BR) = _________.

c. P(BB or RR) = _________.

2. Complete this tree diagram and the table. Show all the possible outcomes of the toss of a coin and one spin on this spinner. The dial is divided into three equal regions.



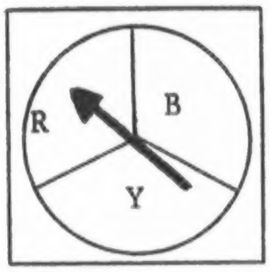
Coin Spinner



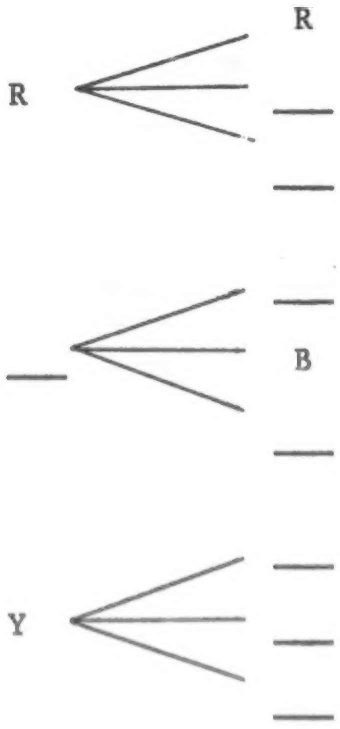
Table

1.
2.
3.
4.
5.
6.

3. Complete this tree diagram and the table to show all the possible outcomes of two spins. The dial is divided into three equal regions.



First Spin Second Spin **Table**



The total number of outcomes is _________.

a. P(RB) = _________.

b. P(YR) = _________.

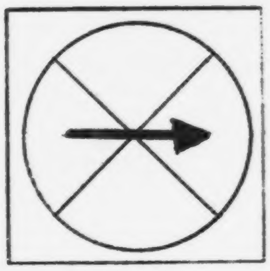
c. P(Orange and Orange) = _________.

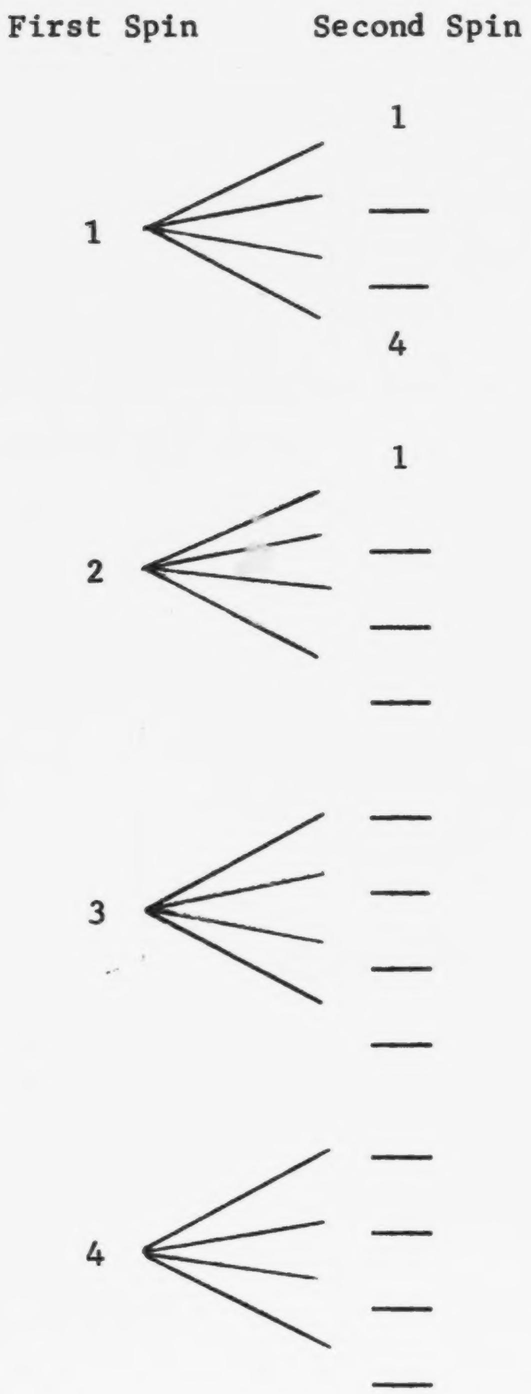
d. P(BY or RR) = _________.

e. P(RY or BY or YY) = _________.

f. How many possible outcomes are there if this spinner is spun three times? _________ (Hint: use tree diagram or observe the pattern)

4. Complete this tree diagram to show all the possible outcomes of two spins with this spinner. The dial is divided into four equal regions.





Table

The total number of outcomes is _____.

a. P(3,4) = _________.

b. P(2,3 or 3,2) = _________.

c. P(Two odd numbers) = _________.

d. P(Three odd numbers) = _________.

e. P(Two odd numbers or two even numbers) = _________.

(Brain Teaser). This spinner is spun four times. How many possible outcomes are there? _________

QUIZ I

Name ________________________________

True or False: Write T (True) of F (False) in the blank at the left of the following problems.

________1. When you toss a coin, you are uncertain what the outcome will be.

________2. When you toss a coin 40 times you will most likely get between 15 and 25 heads.

________3. You have tossed a quarter fairly 50 times. You have gotten 35 heads and 15 tails. If you were going to toss the coin again you are more likely to get a head than a tail.

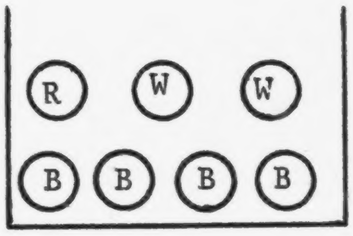
________4. It is possible for two teams to perform the experiment of tossing the same coin 40 times and one team get 16 heads and 24 tails while the other team gets 26 heads and 14 tails.

Multiple Choice

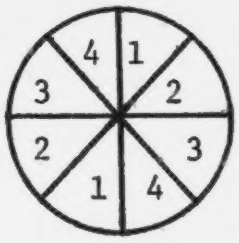
________5. When you toss a coin fairly you are:
(a) more likely to get a head than a tail

(b) more likely to get a tail than a head
(c) a head or a tail is equally likely.

You pick one marble from the box at the right containing red, white and blue marbles.



1. How many possible outcomes are there? ________
2. P(R) = ________
3. P(B) = ________
4. P(R or W) = ________
5. P (Green marble) = ________
6. P(R or W or B) = ________



You spin the spinner at the right one time.
(If it falls on a line you spin it again.)

________1. How many possible outcomes are there?

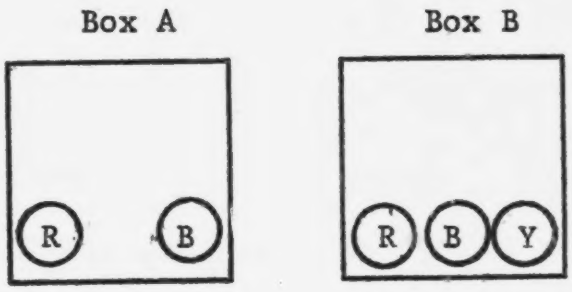
________2. What is the probability of getting a "1"?

________3. What is the probability of an even number?

________4. What is the probability of getting a number less than 6?

________5. What is the probability of getting a number greater than 8?

You pick a marble from Box A and one from Box B.
The marbles are colored red, blue or yellow.
The outcomes are



1. RR
2. RB
3. RY
4. BR
5. BB
6. BY

1. How many outcomes are there? _______

2. P(RR) = _______

3. P(RR or BB) = _______

4. P(R from Box A and any other color from Box B) = _______.

ANALYSIS OF QUIZ I

The quiz (20 items) was given at the end of the introduction to Lesson 6, Part I.

The objectives measured by the quiz were:

1. counting the number of possible outcomes from an experiment (1D and 2D) (3 items)
2. specifying the probability of
    - (a) simple event (2 items)
    - (b) compound event (7 items)
    - (c) certain event (2 items)
    - (d) impossible event (2 items)
3. identifying equally likely outcomes (1 item)
4. identifying the likely bounds on a repeated event (1 item)
5. identifying the uncertainty as to the frequency of the results in carrying out an experiment (1 item)
6. identifying an instance of the law of average (1 item)

The mean on the 20 item quiz was 16.72 and the variance 8.12. 15/25 were at the 90% level or better, 20/25 were at the 80% level or better, and 23/25 were at the 70% level or better. The five people below 80% were subjects "3" - 75%, (He had been absent Friday), "9" - 70% (inconsistent responses), "15" - 70% (problem with probability of certain, impossible and compound events (10)), "20" - 30% (absent for Lesson 4 - she was not using ratios to specify the probability), and "22" - 60% (inconsistent responses).

The item difficulties were:

	1	22/25
page 1	2	19/25
	3	20/25
	4	21/25
	5	24/25
	1	15/25*
page 1	2	24/25
	3	23/25
	4	21/25
	5	25/25
	6	22/25
page 2	1	14/25*
	2	20/25
	3	22/25
	4	22/25
	5	23/25
page 2	1	25/25
	2	22/25
	3	23/25
	4	11/25*

The three starred items had item percentages below 70%. On the first two starred items, p. 1 (1), and p. 2 (1), students specified the number of possible outcomes in terms of an attribute of differentness such as the number of different colors of marbles (3) or the number of different numbers on the spinner in the two respective problems.

At the bottom of page 2 (4), part of the difficulty was with the same problem of wording as was encountered in Lesson 5.

"4. P(R from Box A and any other color from Box B)."
Since the difficulty in the wording "and any other . . ." was discovered at this point 2/6 and 3/6 were both accepted. The wrong responses given for the item were 4/5, 2/5, 4/6 1/6, 5/6, 4/6, 1/6, 1, 4/6, 4/16, 4/5.

The 4 in the numerator probably came from adding the 1Ⓡ from Box A with a Ⓡ, Ⓑ and Ⓨ from Box B. The 5 in the denominator, was probably gotten by adding Ⓡ, Ⓑ and ⓇⒷ, Ⓨ together as 5 objects.

The quiz seemed to indicate that the instruction had been moderately successful particularly for objectives 2a, 2c, 2d and 4, and fairly successful for 6, and 2b.

Objective 1, counting the number of possible outcomes in a one-dimensional space, will need further practice. The same comment also applies to probability problems in 2D involving "and" statements.

Lesson 6, Part I

TUESDAY, MARCH 18

1. Go over homework.
2. Go over problems on the following sheet.
3. Pass out Quiz from yesterday and discuss the results.
4. Tell the students that there will be another quiz similar to Quiz IIB (the last ten problems on the quiz) on Thursday and that they will have another opportunity to be a master.
5. Ask students to bring in games with dice and spinners and also articles that they find concerning probability in newspapers and magazines.

SUPPLEMENT TO LESSON 6

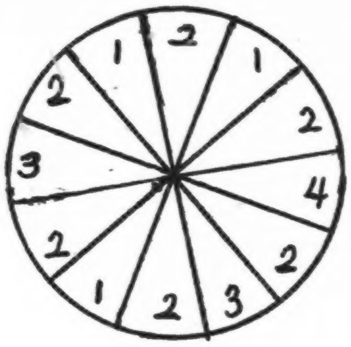
Class Exercise Arising from Analysis of Quiz I

When you toss a die (show class the die)

1. How many possible outcomes are there?
2. P(even number) = ?
3. P(even number or an odd number) = ?

Draw the following spinner on the board. If you spin the spinner below once

1. How many possible outcomes are there?
2. P(2) = ?
3. P(1 or 2) = ?
4. P (even number) = ?



DISCUSSION OF LESSON 6, PART I

Tuesday (3/18)

The teacher went over the homework first. The students corrected their own work.

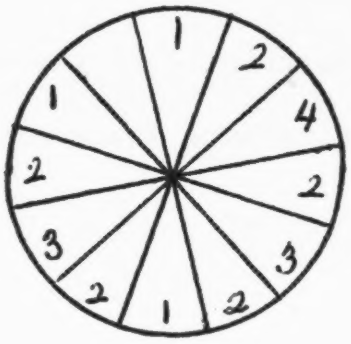
From an analysis of the quiz it was decided that many students (10s/25s) needed more practice using 1-D events. The model of a die was employed first. The teacher showed the class a die. The teacher asked the questions. The students answered the following without any problem.

1. When you toss a die, how many possible outcomes are there?
2. P (even number) = ?
3. P(even or odd number) = ?

The spinner below was then employed because of the students' difficulties with a similar spinner on the quiz. The teacher drew the picture on the board and then asked the following questions.

1. How many possible outcomes are there?
2. P(2) = ?, P(1 or 2) = ?. P(even number) = ?

The class answered the questions without any observable problems.



The quizzes from yesterday were then passed out. The teacher went over all the problems which five or more students had missed. The students argued about the number of possible outcomes when a marble is chosen from a box with red, white and blue marbles. They thought the attribute of color should determine the number of possible outcomes. The teacher then referred them back to the spinner that had just been discussed and for which they had assigned the correct number of possible outcomes. Some were not happy but accepted the point nevertheless.

Because 10s/25s were nonmasters, the class was told they would take another quiz on Thursday over the same concepts. Some of the masters asked if they had to take the quiz also. The teacher told them that they did and that they shouldn't have any problem since they were masters on the first quiz.

The teacher passed out the handout on dice by mistake. The observer reminded her to pass out Handout 2 to allow students practice on using trees. The teacher then passed out Handout 2, Lesson 6. Because of its poor format, the teacher told the class that the blank on the form was for the number of possible outcomes and where to put trees and probability answers.

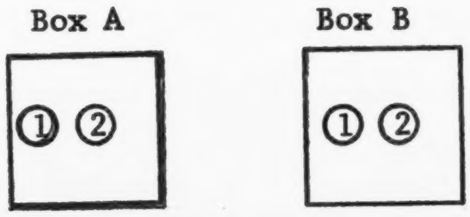
The lesson had called for her to work the first couple of problems of Exercise II together. She chose to allow them to work the problems individually with both of us helping students who had questions. Some students were trying to do the problems without drawing a tree and were making mistakes. The class was instructed by the teacher to draw a tree for each problem. The students were instructed to finish the exercise for tomorrow.

The teacher forgot to remind students to bring in games which use dice or a spinner and articles from the paper or magazines which use probability statements.

After the class the experimenter arranged with the school to have an extra half hour each Wednesday to work with the nonmasters who definitely need more help.

EXERCISE 6.2 (LESSON 6 PART I)

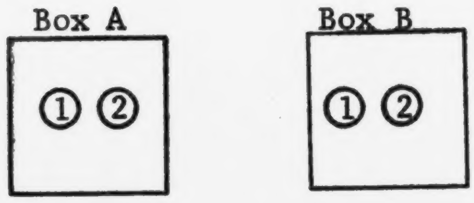
For the following problems you choose a numbered chip from Box A and one from Box B. Find the number of possible outcomes for each problem. Use tree diagrams to arrive at your answer. Show your work to the right of the problem. Place answers to probability questions to the right of the problem.


Box A
①②
Box B
①②

_______ 1.

(a) P(①,①) =

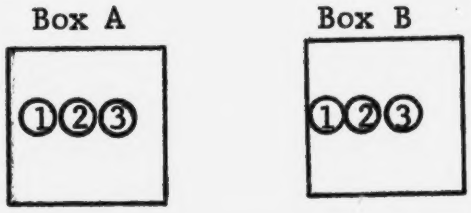
(b) P(①, ② or ②, ①) =


Box A
①②
Box B
①②

_______ 2.

(a) P(①,②) =

(b) P(Sum of the two numbers = 3) =

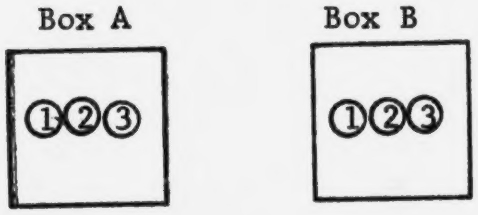

Box A
①②③
Box B
①②③

_______ 3.

(a) P(①,③) =

(b) P(1 , 3 or 1 , 2) =

(c) P(Sum = 4) =

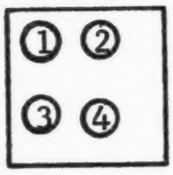

Box A
①②③
Box B
①②③

_______ 4.

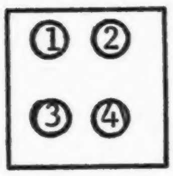
(a) P(①,④) =

(b) P(Sum = 5 or Sum = 6)

Box A



Box B



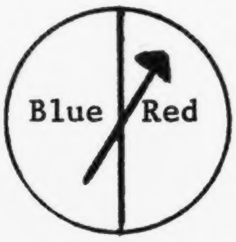
______ 5.

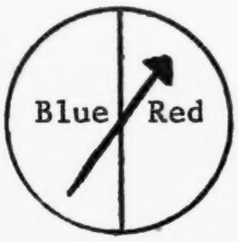
(a) P(①, ④) =
(b) P(Sum = 7) =
(c) P(Sum = 4 or Sum = 5 or Sum = 6) =

For the following problems you spin each of the two spinners in a problem once. Find the number of possible outcomes for each problem. Show the work you did to answer the problem.

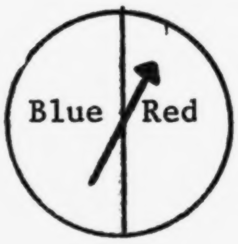
First Spinner Second Spinner

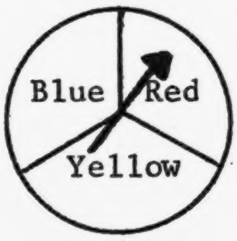
__________ 1.



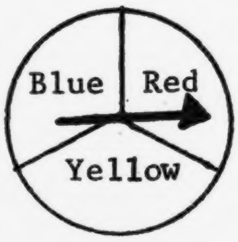


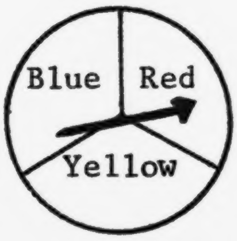
__________ 2.



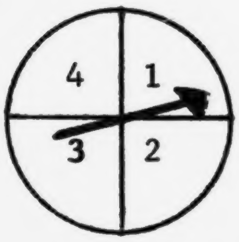

Blue
Red
Yellow

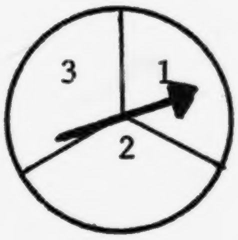
__________ 3.


Blue
Red
Yellow


Blue
Red
Yellow

__________ 4.





In problems 2, 3, and 4 you spin the first spinner twice. Find the number of possible outcomes for each of these.

_______ 2.

_______ 3.

_______ 4.

Lesson 6--Part II*

1. Go over homework.
2. Introduce dice by playing the dice game where you take the sum 5, 6, 7, 8, 9 and they take 2, 3, 4, 10, 11, 12. Point out to them that they have one more sum than you. Have two students mark down on the board 1 tally mark for the appropriate side each time the dice is thrown.

   Go around the room letting each student throw the dice once. At the end ask them why you won--they have one more than you. (You should, since your chances are 24/36 and their's is 12/36.) Ask them if they can tell you how many outcomes there are when the dice are thrown? (Some one may suggest "tree" as a way.)

   Pass out sheets that will help them list outcomes from dice and give them practice in doing problems. Have them do these at their seats.

   Discuss dice problems when they are completed. Look at 4 (h) and (i) in particular and ask them again why they think you won the game at the beginning of the period. (Because 24/36 > 12/36 and thus you have more chances of winning.)

   Have them complete the rest of the problems for tomorrow.

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*Repeated from Lesson 6, Parts I and II.

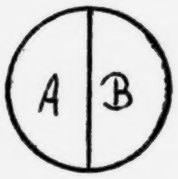
DISCUSSION OF LESSON 6 PART II

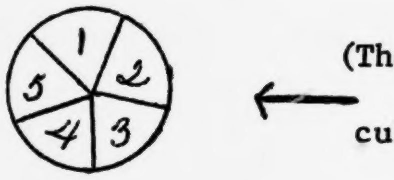
Wednesday (3/19)

The lesson started by the teacher reading the newspaper article about decideding the basketball draft rights to a famous player by tossing a coin. The boys were familiar with the situation.

The homework from yesterday was discussed by the teacher reading the correct answers and having students put two of the problems from the handout on the board to show the tree they used. Problem 5 with chips and Problem 2 with spinners were put on the board. The teacher then asked, "If I spin the first spinner in Problem 2 twice, which other pro-on this page is just like this one?" The teacher explained what was expected for the last three items. Some students were confused by the directions for these.

The teacher then put the following problem on the board for the students to do at their seats. Suppose you spin each of the spinners below one time.





(The teacher had a difficult time drawing this.)

1. How many possible outcomes are there? Draw a tree to find out.
2. From the tree make a list of all the possible outcomes.
3. How many possible outcomes are there?
4. P(A, 5) = ?

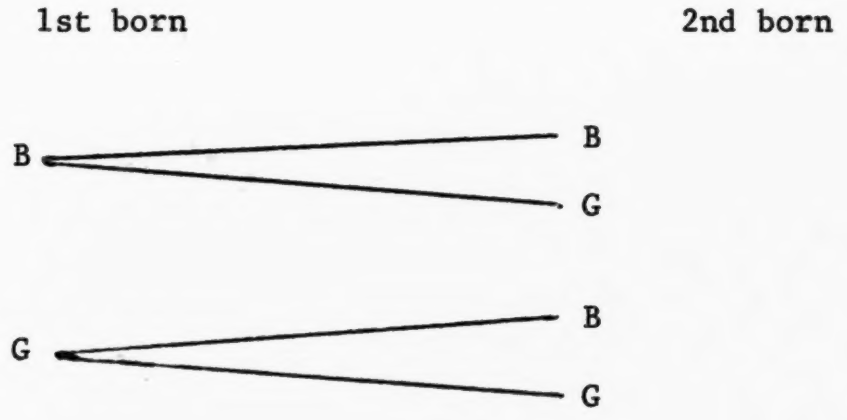
   P(A, even number) = ?

   P(B, any number) = ?

The first boy who finished the problem was chosen to draw the tree on the board.

Many students finished the problem quickly. However, a few were taking a long time. While the teacher helped these, the observer introduced a bonus problem for which they could get extra credit.

The problem was to calculate the probability of their parents having the children they have in their family. The observer did the problem for one child and then two children using a tree in the latter problem.



The observer told them he had a boy 6 and a girl 3. What is P(BG)? They said 1/4.

He told them that if they wished to do this problem for extra credit that they could hand their solution to him for correcting.

The teacher then went over the first problem.

After this she proceeded to do Lesson 6, Part II as planned.

Before the handout was passed out, the first verbal problem from the handout was put on the board and discussed. The handout was then passed out. Most students completed the problems on dice in class. The word problems were briefly discussed and the children were told not to spend too much time on the brain teasers if they had no idea how to do the problem.

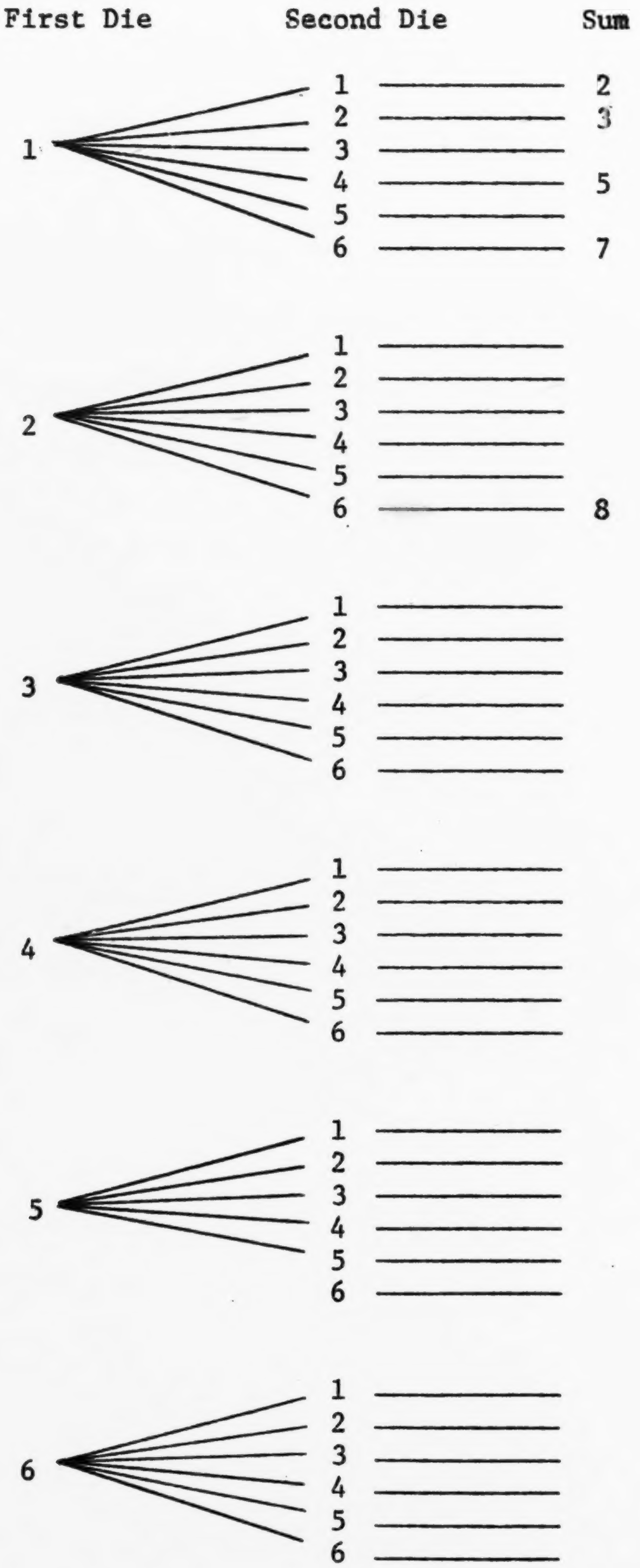
The students were instructed to complete the assignment for tomorrow.

After class the observer helped three boys, subjects 3, 22, and 16 (he had been absent), and the teacher helped two girls, subjects 15 and 20, on one and two-dimensional problems. Actual spinners were used as models. The students were instructed on how to distinguish between problems which needed a tree and those that don't. Questions concerning the number of possible outcomes and the probability of an event in 1D and 2D were made up by the teacher and the observer as they went along.

EXERCISE III

Lesson 6—Part II

Complete this tree diagram to get all the possible outcomes for the sum of the number of dots on the top faces of the dice.


First Die
Second Die
Sum
1
1
2
3
4
5
6
2
3
5
7
2
1
2
3
4
5
6
8
3
1
2
3
4
5
6
4
1
2
3
4
5
6
5
1
2
3
4
5
6
6
1
2
3
4
5
6

Fill in the table below from the tree diagram.

Sum	Number of the Sums	Probability
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
Total		

Lesson 6 Part II

For the following problems, two dice are thrown, and the sum of the faces turning up are recorded.

__________ 1. How many ways can all the sums be gotten?

__________ 2. What is probability of getting the sum 6 (P [6])?

__________ 3. What is the probability that the sum will be either 5 of 9 (P [sum = 50 or sum = 9])?

4. (a) P (sum = 3) = ______

(b) P (sum = 8) = ______

(c) P (sum = 11) = _____

(d) p (sum = 11) = _____

(e) p (sum is greater than 13) = ______

(f) P (sum = 2 or sum = 12) = _____

(g) P (sum = 7 or sum = 11) = _____

(h) P (sum = 2,3,4,10,11, or 12) = _____

(i) p (sum = 5,6,7,8 or 9) = _____

(j) P (sum is less than 15) = _____

_____1. Mary has two skirts, gray and blue, and three blouses, red, light blue and white. How many different skirt and blouse outfits can she wear? List them. (Make a tree diagram of your solution.)

_____2. In Problem 1, if Mary chooses a skirt and blouse without looking, what are the chances that:

_____ a. Mary will wear a gray skirt and a white blouse?

_____ b. Mary will wear a gray skirt (any color of blouse)?

_____ c. Mary will wear a white blouse (any color of skirt)?

_____3. George has four pairs of trousers and six shirts. How many different trouser and shirt combinations can he wear?

_____4. In a certain game, each player is to choose one letter and then one number. There are eight letters, A, B, C, D, E, F, G, and H and seven numbers, 1 through 7. In how many different ways can John choose one letter and one number from these groups? Show by a tree diagram how you could list all of the choices. (You need only complete the first column of the diagram and the part in the second column opposite just one of the elements in the first column.)

_____5. In Problem 4, if a player chooses a letter and a number without looking, what is the probability that he chooses:

_____ a. the letter A and the number 1?

_____ b. the letter A?

_____ c. the number 7?

_____ d. the number 9?

____11. In Problem 10, Janet cannot make up her mind. She decided to make the choice without looking. What are the chances that she chooses:

_____ a. a brown skirt and a yellow blouse?

_____ b. a brown skirt?

_____ c. a gold blouse?

____12. At a picnic, there were the following:

Beverages: milk, iced tea, soft drink.

Salads: bean, tossed, fruit, shrimp

Meats: beef, ham, tuna fish

Vegetables: corn, baked beans, green beans, peas, squash

Desserts: gelatin, cake, ice cream

If you were to choose one and only one item from each of the food categories above, in how many ways could you choose your meal? (Brain teaser)

_____6. Helen is choosing the outfit she wants to wear to a party. She had the choice of three pairs of shoes, four dresses and two hats. How many different outfits does she have to choose from if one outfit consists of a hat, a pair of shoes and a dress?

_____7. At a party you may select one snack and one kind of beverage from the following:

Snack: potato chips, corn chips, popcorn
Soft Drink: cola, orange, grape, cherry

In how many different ways could you select one kind of snack and one flavor of drink.

_____8. In Problem 7, if you choose a snack and one kind of beverage without looking, what is the probability of choosing potato chips and an orange drink?

_____9. Joe's Restaurant has hamburgers advertised:

"Single or double 'burger on bread or bun
With any one of the following: mustard, catsup,
pickle, lettuce, tomato, onion, or plain."

_____ a. In how many different ways can you order hamburgers at Joe's?

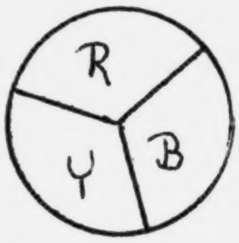
_____ b. In Problem 9 if a boy chooses a hamburger without looking, what is the probability that he will order a double burger on a bun with catsup?

____10. Janet plans to make a skirt and blouse with the skirt a different color from the blouse. She is considering gold, brown, green, and yellow. In how many ways can Janet select colors for one skirt and a blouse?

Lesson 7, Part I

THURSDAY, MARCH 20

1. Go over homework assignment (Exercise III).
2. Give Quiz.
3. Show spinner.



Question: So far we've used two ways to count the number of possible outcomes.

a. We've just counted as when we spin this spinner once.

b. We've made a tree.

How do you know when to use each way of counting the possible outcomes?

4. Do Lesson 7.
5. Pass out exercise on ordering two fractions.

LESSON 7 PART I

Thursday (3/20)

The homework from Wednesday's lesson was discussed first. The problems were corrected in class. As expected, many seemed to have problems doing the verbal problems, particularly the more difficult problems such as 3, 6, 9, 10, and 12.

This took 12 minutes. The quiz was given then and took approximately 15 minutes.

Due to the students' confusion in distinguishing between problems which need a tree and those that don't need a tree, the teacher introduced the following question:

> "So far we've used two ways to count the number of possible outcomes. (a) We've just counted as when we spin this spinner once. (b) We've made a tree. How do you know when to use each way of counting the possible outcomes?"

The teacher then showed the class the spinner which was divided into three equal parts and asked, "If you spin it once, how do you tell how many outcomes there are?" The students answered correctly that you count the number of parts.

"If you spin it twice, how do you count the outcomes?" One student said to just double the first answer. Another said to draw a tree. A tree was drawn by the teacher to show that doubling was not the way to find the number of possible outcomes.

The teacher then stressed that one just counts when he spins once, picks once from a box, etc. One uses a tree when he spins twice, spins two spinners one time each, picks twice from a box, picks one chip from each of two boxes.

This took approximately five minutes. The last 15 minutes were spent on Lesson 7, Part I. The game was played one time by each child with Box B winning more times than Box A.